

A NEW APPROACH TO PHYSICS OF NUCLEI

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Abstract

We employ the QCD sum rules method for description of nucleons in nuclear matter. We show that this approach provides a consistent formalism for solving various problems of nuclear physics. Such nucleon characteristics as the Dirac effective mass m^* and the vector self-energy Σ_V are expressed in terms of the in-medium values of QCD condensates. The values of these parameters at saturation density and the dependence on the baryon density and on the neutron-to-proton density ratio is in agreement with the results, obtained by conventional nuclear physics method. The contributions to m^* and Σ_V are related to observables and do not require phenomenological parameters. The scalar interaction is shown to be determined by the pion–nucleon σ -term. The nonlinear behavior of the scalar condensate may appear to provide a possible mechanism of the saturation. The approach provided reasonable results for renormalization of the axial coupling constant, for the contribution of the strong interactions to the neutron–proton mass difference and for the behavior of the structure functions of the in-medium nucleon. The approach enables to solve the problems which are difficult or inaccessible for conventional nuclear physics methods. The method provides guide-lines for building the nuclear forces. The three-body interactions emerge within the method in a natural way. There rigorous calculation will be possible in framework of self-consistent calculation in nuclear matter of the scalar condensate and of the nucleon effective mass m^* .

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1 Introduction

In the present paper we review our approach to description of a nucleon, placed into nuclear matter. The in-medium characteristics of nucleon can tell us much about the medium itself. On the other hand, the introduction of nuclear matter enables to separate the problems of nucleon interactions from those, connected with individual features of specific nuclei. Thus investigation of nucleon characteristics in nuclear matter is an important step for studies of physics of nuclei.

Until mid 70-th the studies of nuclear matter were based on the nonrelativistic approach. Since the publication of the paper [1] description of the in-medium nucleon was based on Dirac phenomenology. It was successful in describing most of characteristics of nucleons both in nuclear matter and in finite nuclei [2, 3]. In the meson exchange picture the vector and scalar fields correspond to exchange by the vector and scalar mesons between a nucleon and the nucleon of the matter. This picture is called Quantum Hadrodynamics (QHD). In the simplest version (QHD-I) only vector ω and scalar σ mesons are involved. In more complicated versions some other mesons are included [2, 4]. While the studies in framework of the nonrelativistic approach are going on, i.e. the applications of the Nuclear Density Functional Method provided very accurate description of the data [5, 6].

On the other hand, many efforts have been made to improve the QHD. There were many reasons for this. First of all, the QHD itself has several weak points. It is not clear, if the scalar σ meson does exist, the experimental data are controversial. It is rather an effective way of describing the two-pion exchange. The mass of this effective state is about 500 MeV. The mass of vector ω meson is about 780 MeV. Hence, the exchange by theses mesons takes place at the distances, where the nucleon can not be treated as a point-like particle. Thus the QHD inherited the problems of nonrelativistic phenomenology, connected with description of the interaction at small distances. The other weak points are discussed in [7, 8]. Also, it is desirable to match the QHD with the Quantum Chromodynamics (QCD) which is believed to be a true theory of strong interactions. Another reason is that there are various problems in nuclear physics. It is desirable to have an approach, which would enable to calculate:

- The nucleon single-particle potential energy $U(\rho)$, where ρ is the density of the baryon quantum number. This enables one to find the saturation density ρ_0 and the single-particle binding energy $\varepsilon(\rho)$.
- Parameters of interaction with external fields. These are magnetic moments $\mu(\rho)$ and the axial coupling constant $g_A(\rho)$. The latter is important for understanding of the chiral properties of the matter.

- Neutron–proton mass splitting in isotope-symmetric matter. It was observed for a number of nuclei, being known as the Nolen–Schiffer anomaly.
- Structure functions of the deep inelastic scattering. They describe the internal structure of nucleons. Investigation of the latter is important for construction of the quark models of nucleons, for studies of the confinement, etc. The reasons for some of the in-medium modifications of the structure functions are still obscure. They become a subject for discussions from time to time.
- The single particle potential energy for hyperons in nuclear matter. Can a system of hyperons be stable? In other words, can a strange matter exist?

As we said earlier, the first problem was solved in framework of Dirac phenomenology [1, 2] for the values of density close to the saturation value. The nucleon was considered as a relativistic particle, moving in superposition of vector and scalar fields V_μ and Φ . In the rest frame of the matter $v_\mu = V_0\delta_{\mu 0}$. The dynamics of the nucleon is described by equation

$$(\hat{q} - \hat{V})\psi = (m + \Phi)\psi, \quad (1)$$

with $\hat{A} = A_\mu\gamma^\mu$, $q_\mu = -i\partial_\mu$. In nuclear matter the fields V and Φ depend only on the density ρ , and do not depend on the space coordinates. The values of the fields are adjusted to reproduce either the data on nucleon–nucleon scattering or the nuclear data – see, e.g. [3]. However, each of the other listed problems requires additional improvements of the QHD. Turning to the other problems from the above list, note that the axial coupling constant changes due to polarization of medium by pions [9], with a crucial role of the delta-isobar excitations [10]. Thus, in order to solve the second problem one should introduce additional degrees of freedom into the QHD. Also, the third problem requires introduction of the nuclear forces, which break the isospin invariance [11]. Finally, the fourth problem is just inaccessible for traditional methods of nuclear physics.

There were several attempts to combine the QHD and the quark structure of nucleons. The nucleon-nucleon forces, constructed within such models enabled to reproduce semi-quantitatively or quantitatively the nucleon characteristics in nuclear matter. In the Quark–Meson Coupling (QMC) model [12] the nucleon was considered as a three-quark system in a bag. The quarks were directly coupled to σ and ω mesons. The values of the nucleon effective mass m^* and of the in-medium coupling constant g_A appeared to be somewhat smaller than in QHD [13]. This model is employed nowadays as well [14].

In another class of works the short-range interaction between the nucleons was treated as interaction between their quarks. The latter were described in framework of some QCD motivated models. The long-range NN interaction was described in terms of nucleons and pions. These works were reviewed in [15].

During the two latest decades much work was done on development of the Effective Field Theory (EFT). The starting point is the most general Lagrangian, which includes nucleons and pions as the degrees of freedom and respects all the symmetries of QCD, i.e. the Lorentz invariance, chiral symmetry, etc. [16]. The applications of the EFT is usually combined with the expansion in powers of the pion mass (Chiral Perturbation Theory). Expansion in powers

of low momenta is also usually carried out. The works, based on the EFT are reviewed in [17], for some of recent developments - see [18]. The EFT approach does not touch the quark degrees of freedom of the nucleons.

All the traditional nuclear physics approaches face difficulties in attempts to describe the nucleon-nucleon (NN) interaction at small distances. On the other hand, the Quantum Chromodynamics (QCD) is believed to be a true theory of strong interactions. It has many unsolved problems at large distances. However, it becomes increasingly simple at small distances due to the asymptotic freedom. The interaction of quarks and gluons, which are the ingredients of QCD can be treated perturbatively. This is known as the asymptotic freedom [19]. It is tempting to use this feature of QCD in building the nucleon forces. One should take into account, however, that due to spontaneous breakdown of the chiral symmetry of the QCD, the vacuum expectations of some QCD operators (condensates) have nonzero values.

In medium with a nonzero value of the density of the baryon quantum number the condensates change their values. Also, some other condensates, which vanish in vacuum, obtain nonzero values.

The QCD sum rules (SR) method describes the vacuum hadron parameters basing on the quark dynamics at short distances, where the asymptotic freedom works. In other words, the dynamics of the quark system at the distances of the order of the confinement radius is described basing on that at small distances, where it is determined by the QCD condensates. Thus, the SR method enables to describe the hadron parameters in terms of the QCD condensates.

The approach was worked out in [20], where it was used for mesons. It is based on the dispersion relation for the function, describing the system which carries the quantum numbers of the hadron. The SR method was successfully applied for nucleons in vacuum [21], describing all their static and some of dynamical characteristics – see [22, 23] for a review. Thus it looks reasonable to try to apply the QCD SR method for the description of nucleons in nuclear matter. It was suggested in [24, 25, 26] that the parameters of nucleon in nuclear matter can be expressed in terms of the in-medium values of QCD condensates. In medium with a nonzero value of the density of the baryon quantum number the condensates change their values. Also, some other condensates, which vanish in vacuum, obtain nonzero values.

The generalization of the SR method for the case of finite densities was not straightforward. One of the main problems was the choice of variables, which enabled to separate the singularities connected with the in-medium nucleon from those connected with the medium itself. This was done in [24]–[26].

It was found also in these papers that the nucleon characteristics (effective mass m^* and the vector self-energy Σ_V) can be presented in terms of the vector and scalar condensates

$$v(\rho) = \langle M | \sum_i \bar{q}^i \gamma_0 q^i | M \rangle; \quad \kappa(\rho) = \langle M | \sum_i \bar{q}^i q^i | M \rangle. \quad (2)$$

Here ρ and $\langle M |$ are the density and vector of the ground state of the matter, q^i is the quark field, the summation over flavors $i = u, d$ is carried out. The vector condensate is written in the rest frame of the matter. Due to conservation of the vector current the vector condensate is a linear function of ρ

$$v(\rho) = v_N \rho; \quad v_N = \langle N | \sum_i \bar{q}^i \gamma_0 q^i | N \rangle = 3 \quad (3)$$

is just the number of valence quarks in a nucleon. The scalar condensate can be represented as [24, 25]

$$\kappa(\rho) = \kappa(0) + \kappa_N \rho + S(\rho), \quad \kappa_N = \langle N | \sum_i \bar{q}^i q^i | N \rangle, \quad (4)$$

with $S(\rho)$ caused by interaction of the nucleons of the matter. Since κ_N can be expressed in terms of the pion–nucleon σ term [27], while the latter is related to observables [28, 29], one can obtain the values of the nucleon parameters at least in the gas approximation. Since $S(\rho)$ is small for the densities, close to the saturation point (see below), the values obtained in such approach are close to the physical ones. They appeared to be $\Sigma_V \approx 200$ MeV, and $m^* - m \approx -300$ MeV close to the saturation point. Thus, including only the condensates of the lowest dimension and neglecting the radiative corrections, we found that the QCD SR method reproduce the main features of the QHD [1].

However, the role of the condensates of higher dimension remained obscure. The contribution of the gluon condensate $\langle M | \frac{\alpha_s}{\pi} G^{a\mu\nu} G_{\mu\nu}^a | M \rangle$ is rather small. However, an estimation for the value of the four-quark condensate $\langle M | \bar{q}q\bar{q}q | M \rangle$, which suggests itself, destroys the agreement with the Walecka model. Also, the lowest order radiative corrections are numerically large. This took place for the vacuum as well. This caused doubts in possibility to expand the SR method for the case of finite densities.

The four-quark condensates are the most important among those of the higher dimension. Their calculation requires some model assumption on the quark structure of nucleon. We employed the Perturbative Chiral Quark Model (PCQM), suggested originally in [30]. We calculated these condensates [31] and found that previous naive estimation of its contribution was wrong, due to some cancelations which take place in any reasonable model. We demonstrated that the SR method provides the dependence of the nucleon characteristics m^* and Σ_V on the density ρ and on the neutron-to-proton density ratio [32, 33], which is consistent with the results obtained by using traditional nuclear physics methods.

We analyzed the role of radiative corrections for the nucleon SR in vacuum and demonstrated that their influence on the value of the nucleon mass is small [34]. Also, in nuclear matter the radiative corrections do not change much the nucleon characteristics m^* and Σ_V [35].

We found that the nonlinear contribution to the scalar condensate $S(\rho)$ is determined mainly by the pion contribution to the self-energy of the nucleon of the matter. Simple estimations show that this term may provide a saturation mechanism in our approach [36]. However, a rigorous treatment requires renormalization of the pion propagator by the particle-hole excitations. The renormalized pion propagator depends on the nucleon effective mass m^* and on the in-medium value of the pion–nucleon coupling constant g_s . The latter can be obtained by the SR method. This brings us to a self-consistent scenario. At the present level of our knowledge it requires some more phenomenological assumptions [37].

Note also, that the finite density SR method provided reasonable results for the axial coupling constant [38], for the neutron–proton mass splitting [39] and for the difference between the deep inelastic structure of nucleus and that of sum of those of free nucleons [40]. However, in these calculations only the condensates of the lowest dimensions have been included.

Now we give the details.

2 Nucleon QCD sum rules in vacuum

This approach is described in details in many publications. Earlier papers are reviewed in [41]. The nowadays state of art is presented in [22] and [23], see also the book [19]. However, to make the text self-consistent we recall the main points of the approach. We emphasize the points, which we shall need for the extension of the SR method for the case of finite density of the baryon quantum number.

2.1 General ideas

The nucleon QCD sum rules succeeded in describing of the nucleon characteristics in vacuum in terms of the vacuum expectation values of the products of quark or (and) gluon operators (QCD condensates) [7, 8]. This approach is based on the dispersion relation for the function

$$\Pi_0(q) = \hat{q}\Pi_0^q(q^2) + I\Pi_0^I(q^2),$$

describing the propagation of the system which carries the quantum numbers of the proton, I is the unit 4×4 matrix. In the simplest form the dispersion relations are

$$\Pi_0^i(q^2) = \frac{1}{\pi} \int dk^2 \frac{\text{Im}\Pi_0^i(k^2)}{k^2 - q^2}; \quad i = q, I. \quad (5)$$

As we shall see below, we do not need to worry about possible subtractions.

In quantum mechanics $\Pi_0(q^2)$ is just the proton propagator. In the field theory different degrees of freedom are important in different regions of the value of q^2 .

One can consider the proton as a system of three strongly interacting quarks. Due to asymptotic freedom of QCD

the description becomes increasingly simple at $q^2 \rightarrow -\infty$. This means that at $q^2 \rightarrow -\infty$ the function $\Pi_0(q^2)$ can be presented as a power series of q^{-2} and of the QCD coupling constant α_s . The coefficients of the expansion in powers of q^{-2} are the QCD condensates. Such presentation known as the Operator Product Expansion (OPE) [42] provides the perturbative expansion of the short distance effects, while the nonperturbative physics is contained in the condensates. In QCD SR approach the left-hand side (LHS) of Eq.(5) is considered at $q^2 \rightarrow -\infty$, and several lowest order terms of the OPE are included. In this and in the next Subsections we neglect the radiative corrections, i.e. we do not include interactions and self-interactions (self-energy insertions) of the quarks, putting $\alpha_s = 0$.

Turning to the right-hand side (RHS) of Eq. (5) note that $\text{Im}\Pi_0(k^2) = 0$ at $k^2 < m^2$ with m being the position of the lowest lying pole, i.e. m is the proton mass. There are the other singularities at larger values of k^2 . These are the cuts corresponding to the systems “proton+pions”, the pole N(1440), etc. The next to leading singularity is the physical branching point $k^2 = W_{phys}^2$, and one can write

$$\text{Im}\Pi_0^i(k^2) = \lambda_N^2 \xi^i \delta(k^2 - m^2) + f^i(k^2) \theta(k^2 - W_{phys}^2); \quad i = q, I, \quad (6)$$

with $\xi^q = 1$, $\xi^I = m$; λ_N^2 – the residue at the pole. Now Eq. (5) takes the form

$$\Pi_0^{OPE}(q^2) = \frac{\lambda_N^2 \xi^i}{m^2 - q^2} + \frac{1}{\pi} \int_{W_{phys}^2}^{\infty} dk^2 \frac{f^i(k^2)}{k^2 - q^2}; \quad i = q, I. \quad (7)$$

The upper index OPE means that several lowest OPE terms are included.

The SR approach is focused on studies of the lowest state. Thus we keep the first term on the RHS of Eq. (7) and try to write approximate expression for the contribution of the higher states. The detailed structure of the spectral function $f(k^2)$ in the second term on the RHS of Eq. (7) cannot be obtained by the SR method. However, at $k^2 \gg |q^2|$

$$f^i(k^2) = \frac{\Delta \Pi_0^i{}^{OPE}(k^2)}{2i}, \quad (8)$$

with Δ denoting the discontinuity.

The standard ansatz consists in extrapolation of Eq. (8) to the lower values of k^2 , assuming that the cut starts at certain unknown point W^2 . In other words

$$\frac{1}{\pi} \int_{W_{phys}^2}^{\infty} dk^2 \frac{f^i(k^2)}{k^2 - q^2} = \frac{1}{2\pi i} \int_{W^2}^{\infty} dk^2 \frac{\Delta \Pi_0^i{}^{OPE}(k^2)}{k^2 - q^2},$$

and thus Eq. (5) takes the form

$$\Pi_0^i{}^{OPE}(q^2) = \frac{\lambda_N^2 \xi^i}{m^2 - q^2} + \frac{1}{2\pi i} \int_{W^2}^{\infty} dk^2 \frac{\Delta \Pi_0^i{}^{OPE}(k^2)}{k^2 - q^2}; \quad i = q, I. \quad (9)$$

Recall that $\xi^q = 1$, $\xi^I = m$.

This “pole+continuum” presentation of the RHS makes sense only if its first term, treated exactly is larger than the second term, which approximates the higher states. The position of the lowest pole m , its residue λ^2 and the continuum threshold W^2 are the unknowns in Eqs. (9).

In the next step one usually applies the Borel transform. It is defined as

$$BF(Q^2) = \lim_{Q^2, n \rightarrow \infty} = \frac{(Q^2)^{n+1}}{n!} \left(-\frac{d}{dQ^2} \right)^n F(Q^2) \equiv \tilde{F}(M^2);$$

$$Q^2 = -q^2, \quad M^2 = Q^2/n, \quad (10)$$

converting a function of q^2 into the Borel transformed function of the Borel mass M^2 . The reasons for applying the transform are

- Since $B(Q^2)^k = 0$ for any integer k , it kills the polynomials of q^2 . Hence, it eliminates the divergent terms in the function Π_0^i . This explains, why we wrote the dispersion relation (5) without subtractions.
- It emphasizes the contribution of the lowest state, since

$$B \frac{1}{Q^2 + m^2} = e^{(-m^2/M^2)}.$$

- It improves the convergence of the OPE series, since

$$B[(Q^2)^{-n}] = \frac{(M^2)^{1-n}}{(n-1)!}.$$

The Borel transformed SR take the form

$$\tilde{\Pi}_0^i{}^{OPE}(M^2) = \lambda_N^2 \xi^i e^{(-m^2/M^2)} + \frac{1}{2\pi i} \int_{W^2}^{\infty} dk^2 \Delta \Pi_0^i{}^{OPE}(k^2) e^{(-k^2/M^2)}. \quad (11)$$

If both LHS and RHS of Eq. (11) were calculated exactly, the relation would be independent on M^2 . However, certain approximations are made on both sides. The analytical dependence of both sides on M^2 is quite different. The OPE on the LHS becomes increasingly true at large M^2 . The accuracy of the “pole+continuum” model used for the RHS increases at small M^2 . An important assumption is that there is an interval of the values of M^2 , where the two sides have a good overlap, approximating also the true functions $\tilde{\Pi}_0^i(M^2)$.

Thus our task is to find the region of the values of M^2 , where the overlap can be achieved and to obtain the characteristics of the lowest state m and λ^2 and the value of the continuum threshold W^2 .

2.2 Explicit form of the SR equations

The general definition of the function $\Pi_0(q^2)$, which is sometimes called “correlator” or the “correlation function” is

$$\Pi_0(q^2) = i \int d^4x e^{i(q \cdot x)} \langle 0 | T[j(x) \bar{j}(0)] | 0 \rangle, \quad (12)$$

where j is the local operator with the proton quantum numbers. It is often refereed to as “current”. It was shown in [36] that there are three independent currents

$$j_1 = (u_a^T C \gamma_\mu u_b) \gamma_5 \gamma^\mu d_c \varepsilon^{abc}, \quad j_2 = (u_a^T C \sigma_{\mu,\nu} u_b) \gamma_5 \sigma^{\mu,\nu} d_c \varepsilon^{abc}, \quad (13)$$

$$j_{3\mu} = \left[(u_a^T C \gamma_\mu u_b) \gamma_5 d_c - (u_a^T C \gamma_\mu d_b) \gamma_5 u_c \right] \varepsilon^{abc}.$$

Here a, b, c are the color indices, C is the charge conjugation matrix, while T denotes the transpose in the Dirac space. It was shown in [43] that the operators j_2 and $j_{3\mu}$ provide strong admixtures of the states with negative parity and of the states with spin 3/2 correspondingly. Thus, the calculations with the operator j_1 are most convincing. We shall assume $j = j_1$ in further studies.

Expansion of the matrix element on the RHS of Eq. (12) in powers of x^2 corresponds to expansion of its LHS in powers of q^{-2} . In the lowest orders of x^2 expansion the T product on the RHS of Eq. (12) can be written in terms of those of two quark operators. The latter can be written, following the Wick theorem

$$\langle 0 | T[q_i^a(x) \bar{q}_j^b(0)] | 0 \rangle = g_q(x) - \frac{1}{12} \sum_A \Gamma_{ij}^A \delta^{ab} \langle 0 | \bar{q} \Gamma^A q | 0 \rangle + O(x^2). \quad (14)$$

Here

$$g_q(x) = \frac{i}{2\pi^2} \frac{(\hat{x} - im_q x^2/2)_{ij}}{x^4} \delta^{ab} \quad (15)$$

is the free quark propagator, m_q is the quark mass, Γ^A are the basic 4×4 matrices with the scalar, pseudoscalar, vector, pseudovector and tensor structures, i.e. $I, \gamma_5, \gamma_\alpha, \gamma_5 \gamma_\alpha$ and $\sigma_{\alpha\beta}$. For

the fields which respect the chiral invariance all the expectation values on the RHS of Eq. (14) vanish. However in the QCD the expectation value $\langle 0|\bar{q}q|0\rangle$ ($\Gamma_A = I$) has a nonzero value. All the other condensates $\langle 0|\bar{q}\Gamma^A q|0\rangle$ vanish due to invariance of vacuum. Thus

$$\langle 0|T[q_i^a(x)\bar{q}_j^b(0)]|0\rangle = g_q(x) - \frac{1}{12}I_{ij}\delta^{ab}\langle 0|\bar{q}q|0\rangle + O(x^2). \quad (16)$$

Now we present

$$\Pi_0^q(q^2) = \sum_n A_n; \quad \Pi_0^I(q^2) = \sum_n B_n,$$

with n denoting the dimension of the condensate, contributing to the term A_n or B_n .

If all expectation values of the T products of the quark fields are described by the free propagators (15), we find the leading OPE contribution A_0 to the structure $\Pi_0^q(q^2)$, corresponding to the free three-quark loop. If one of the quark pairs is described by the second term on the RHS of Eq. (14), while the others are given by the first term, we find the leading contribution B_3 to the structure $\Pi_0^I(q^2)$. In this case two quarks form a free loop, while the current exchanges by a quark-antiquark pair with vacuum.

Direct calculation provides [21]

$$A_0 = -\frac{q^4 \ln(-q^2/L_q^2)}{64\pi^4}; \quad B_3 = \frac{\langle 0|\bar{d}d|0\rangle q^2 \ln(-q^2/L_q^2)}{4\pi^2}. \quad (17)$$

Here L_q is the cutoff of the integral over x in Eq. (12). Its value is not important, since the terms containing L_q will be eliminated by the Borel transform.

Going beyond the single-particle presentation (14) one finds also the next to leading OPE terms. For example, the contribution A_4 is due to the lowest order interaction of the quark system with the gluon condensate

$$\langle 0|\frac{\alpha_s}{\pi}G^{a\mu\nu}G_{\mu\nu}^a|0\rangle = -2\langle 0|\frac{\alpha_s}{\pi}(E^2 - B^2)|0\rangle,$$

with E and B the color-electric and color-magnetic fields. This condensate also has a nonzero value only due to the violation of the chiral symmetry in the ground state of the QCD. The higher terms contain the four-quark condensate

$$A_6 = -\frac{2\langle 0|\bar{q}q\bar{q}q|0\rangle}{3q^2} \quad (18)$$

Finally, the Borel-transformed sum rules can be written as

$$\mathcal{L}_0^q(M^2, W^2) = R^q(M^2); \quad \mathcal{L}_0^I(M^2, W^2) = R^I(M^2). \quad (19)$$

Here

$$R^q(M^2) = \lambda^2 e^{-m^2/M^2}; \quad R^I(M^2) = m\lambda^2 e^{-m^2/M^2}, \quad (20)$$

with $\lambda^2 = 32\pi^4\lambda_N^2$. The factor $32\pi^4$ is introduced in order to deal with the values of the order of unity (in GeV units). The contribution of continuum is moved to the LHS of Eqs. (19) (see Eq. (22) below). Following [21, 44] we can write

$$\mathcal{L}^q = \tilde{A}_0(M^2, W^2) + \tilde{A}_4(m^2, W^2) + \tilde{A}_6(M^2); \quad \mathcal{L}^I = \tilde{B}_3(M^2, W^2) + \tilde{B}_7(M^2). \quad (21)$$

The terms \tilde{A}_n and \tilde{B}_n are the Borel transforms of the contributions A_n and B_n to the functions $\Pi_0^{q,I}(q^2)$, with subtraction of the corresponding contributions of continuum. Recall that the lower index denotes the dimension of the condensate. The terms, proportional to condensates of higher dimension A_6 and B_7 do not contain the logarithmic loops and thus do not contribute to continuum. Actually the calculations are carried out in the chiral limit $m_q = 0$. The explicit form of the contributions is [21, 44]

$$\begin{aligned} A_0 &= M^6 E_2 \left(\frac{W^2}{M^2} \right); \quad A_4 = \frac{b M^2 E_0 \left(\frac{W^2}{M^2} \right)}{4}; \quad A_6 = \frac{4}{3} a^2; \\ B_3 &= 2a M^4 E_1 \left(\frac{W^2}{M^2} \right); \quad B_7 = -\frac{ab}{12}. \end{aligned} \quad (22)$$

Here

$$E_0(x) = 1 - e^{-x}, \quad E_1(x) = 1 - (1+x)e^{-x}, \quad E_2(x) = 1 - (1+x+x^2/2)e^{-x}, \quad (23)$$

while

$$a = -(2\pi)^2 \langle 0 | \bar{q}q | 0 \rangle; \quad b = (2\pi)^2 \langle 0 | \frac{\alpha_s}{\pi} G^{a\mu\nu} G_{\mu\nu}^a | 0 \rangle. \quad (24)$$

The contributions are illustrated by Fig. 1. Note that we provided these equations mostly as illustration of the main ideas and did not include several numerically not very important terms.

The term \tilde{A}_6 presents the contribution of the four-quark condensates $\langle 0 | \bar{q}\Gamma^A q \bar{q}\Gamma^A q | 0 \rangle$, which, generally speaking, obtain nonzero values for all structures Γ_A . It is evaluated under the factorization approximation [6]

$$\langle 0 | \bar{q}\Gamma^A q \bar{q}\Gamma^A q | 0 \rangle = \frac{1}{16} (\langle 0 | \bar{q}q | 0 \rangle) \left[(\text{Tr} \Gamma_A)^2 - \frac{1}{3} \text{Tr} (\Gamma_A^2) \right].$$

One can find numerical values of the main QCD condensates presented by Eq. (24). There is the well known Gell-Mann–Oakes–Renner relation (GMOR) for the scalar condensate [45]

$$\langle 0 | \bar{u}u + \bar{d}d | 0 \rangle = -\frac{2f_\pi^2 m_\pi^2}{m_u + m_d}. \quad (25)$$

Here f_π and m_π are the decay constant and the mass of the π meson, m_u and m_d are the current masses of u and d quarks. Its numerical value is $\langle 0 | \bar{u}u + \bar{d}d | 0 \rangle = 2(-240 \text{ MeV})^3$. The value of the gluon condensate $\langle 0 | \frac{\alpha_s}{\pi} G^{a\mu\nu} G_{\mu\nu}^a | 0 \rangle \approx (0.33 \text{ GeV})^4$ was extracted from the analysis of leptonic decay of ρ and ϕ mesons [46]. This data were supported by the QCD sum rules analysis of charmonium spectrum [20].

Now one must find the set of parameters m, λ^2, W^2 , which insure the most accurate approximation of $\mathcal{L}^i(M^2)$ by the functions $R^i(M^2)$ and also the interval of the values of M^2 , where this can take place. The set of parameters m, λ^2, W^2 , which minimize the function

$$\chi^2(m, \lambda^2, W^2) = \sum_j \sum_{i=q,I} \left(\frac{\mathcal{L}^i(M_j^2) - R^i(M_j^2)}{\mathcal{L}^i(M_j^2)} \right)^2, \quad (26)$$

will be referred to as a solution of Eqs. (11).

The appropriate interval

$$0.8 \text{ GeV}^2 < M^2 < 1.4 \text{ GeV}^2 \quad (27)$$

and the values of nucleon parameters

$$m = 0.93 \text{ GeV}; \quad \lambda^2 = 1.8 \text{ GeV}^6; \quad W^2 = 2.1 \text{ GeV}^2 \quad (28)$$

were found in [21, 44]. If we fix $m = 0.94 \text{ GeV}$, the SR provide

$$\lambda^2 = 2.0 \text{ GeV}^6, \quad W^2 = 2.2 \text{ GeV}^2. \quad (29)$$

These values will be employed in the present paper.

It was shown also in [21] that the nucleon mass vanishes if there is no spontaneous breakdown of the chiral symmetry, e.g. if $\langle 0 | \bar{q}q | 0 \rangle = 0$. Numerically [21, 44]

$$m^3 \approx -2(2\pi)^2 \langle 0 | \bar{q}q | 0 \rangle. \quad (30)$$

In the QCD SR approach the mass of the nucleon is formed due to exchange by quarks between the nucleon and vacuum.

The role of instantons in QCD SR was analyzed in [47], [48]. It was shown that for the current j_1 the contribution of instantons vanishes for the \hat{q} structure of the SR. In the scalar structure instantons form a factor, which change the LHS of SR by about 15%. Following [22] we include the contribution into uncertainties of the numerical value of the vacuum expectation value $\langle 0 | \bar{q}q | 0 \rangle$. However, a rigorous analysis requires investigation of the M^2 dependence of the contribution.

Note that the solutions (28), (29) are not stable with respect to modification of the values of the condensates [49]. Even for the small changes the absolute minimum of the RHS of Eqs. (19) is provided by another solution, i.e. $m = 0.6 \text{ GeV}$, $\lambda^2 = 0.79 \text{ GeV}^6$, $W^2 = 1.0 \text{ GeV}^2$ [50]. We treat this solution as an unphysical one, since the contribution of the continuum exceeds more than twice that of the lowest pole. This contradicts the key assumption of the “pole+continuum” model for the spectrum – see Eq. (9).

The nucleon SR with another form of nuclear current were obtained in [51]. In [52] it was used also for the description of delta isobars. Further we shall mention some other applications.

2.3 Inclusion of radiative corrections

A typical radiative correction is shown in Fig. 2. In the analysis, carried out in [21, 44] the most important radiative corrections of the order $\alpha_s \ln Q^2$ have been included. These contributions were summed to all orders of $(\alpha_s \ln Q^2)^n$. This is called the Leading Logarithmic Approximation. The LLA corrections are expressed in terms of the factor [53]

$$L(Q^2) = \frac{\ln Q^2 / \Lambda^2}{\ln \mu^2 / \Lambda^2}, \quad (31)$$

where $\Lambda \approx 150 \text{ MeV}$ is the QCD scale, while μ is the normalization point, the standard choice is $\mu = 500 \text{ MeV}$.

$$A_0^r = A_0 / L^{4/9}; \quad B_3^r = B_3. \quad (32)$$

Here the upper index r indicates inclusion of the radiative corrections.

The radiative corrections to the OPE terms of the function Π_0 have been calculated beyond the LLA in the lowest order of the α_s expansion [54] (see also [55]). The results are

$$A_0^r = A_0 \left(1 + \frac{71}{12} \frac{\alpha_s}{\pi} - \frac{1}{2} \frac{\alpha_s}{\pi} \ln \frac{Q^2}{\mu^2} \right); \quad A_6^r = A_6 \left(1 - \frac{5}{6} \frac{\alpha_s}{\pi} - \frac{1}{3} \frac{\alpha_s}{\pi} \ln \frac{Q^2}{\mu^2} \right); \quad (33)$$

$$B_3^r = B_3 \left(1 + \frac{3}{2} \frac{\alpha_s}{\pi} \right).$$

The running coupling constant in the one-loop approximation, with inclusion of three lightest quarks is

$$\alpha_s(k^2) = \frac{4\pi}{9 \ln k^2 / \mu^2}. \quad (34)$$

The calculations in [54, 55] have been carried out for $\alpha_s = \text{const}$. Since the linear momenta in the loops, corresponding to radiative corrections are of the order q , it is reasonable to assume that in Eq. (34) $\alpha_s = \alpha_s(Q^2)$, converting to $\alpha_s(M^2 \approx 1 \text{ GeV}^2)$. Assuming $\alpha_s(1 \text{ GeV}^2) \approx 0.37$ (somewhat larger values are often used nowadays [56]) we find that the radiative correction to the contribution A_0 changes its value by about 50%. This “uncomfortably large” correction was often claimed as the most weak point of the SR approach [57].

In [34] we investigated the influence of radiative corrections on the values of characteristics, obtained in framework of the Borel transformed nucleon SR in vacuum. We demonstrated that inclusion of the radiative corrections in various ways (the radiative corrections are totally neglected, are included in LLA, are taken into account beyond the LLA in the lowest order) alter the value of nucleon mass m by about 5%. The radiative corrections modify mainly the value of the nucleon residue.

Note also that inclusion of the radiative corrections diminishes the role of the unphysical solution, mentioned above. Once they are included, minimization of the function χ^2 defined by Eq. (26) is provided by the physical solution in a broader interval of the values of the condensates [49, 50].

3 QCD sum rules in nuclear matter

3.1 Choice of the variables

Now we shall try to use the SR approach for calculation of the nucleon parameters in nuclear matter. The propagation of the system which has a four-momentum q and carries the quantum numbers of the proton is determined by the equation

$$\Pi_m = i \int d^4x e^{i(q \cdot x)} \Xi(x); \quad \Xi(x) = \langle M | T[j(x) \bar{j}(0)] | M \rangle, \quad (35)$$

with $|M\rangle$ the ground state of the nuclear matter. It is just an analog of Eq. (12). We consider nuclear matter as a system of A nucleons with momenta p_i , introducing

$$P = \frac{\sum p_i}{A}. \quad (36)$$

In the rest frame of the matter $P \approx (m, 0)$.

The spectrum of the function $\Pi_m(q, P)$ is much more complicated than that of the vacuum function $\Pi_0(q^2)$. Our main task is to separate the singularities connected with the nucleon in the matter from those connected with the matter itself. Include in the first step only the two-nucleon interactions. The singularities connected with the matter manifest themselves as singularities in variable $s = (P + q)^2$. Thus the separation can be done by considering $\Pi_m(q^2, s)$ and keeping $s = \text{const}$. Thus we consider the dispersion relations

$$\Pi_m^i(q^2, s) = \frac{1}{\pi} \int dk^2 \frac{\text{Im}\Pi_m^i(k^2, s)}{k^2 - q^2} \quad (37)$$

for the three structures ($i = q, P, I$) of the function

$$\Pi_m(q^2, s) = \hat{q}\Pi_m^q(q^2, s) + \hat{P}\Pi_m^P(q^2, s) + I\Pi_m^I(q^2, s). \quad (38)$$

Each contribution Π^i can be viewed as the sum of the vacuum term Π_0^i and that provided by the nucleons of the matter Π_ρ^i

$$\Pi_m^i = \Pi_0^i + \Pi_\rho^i; \quad \Pi_0^P = 0. \quad (39)$$

This notation will be used also for the other functions.

We clarify the value of s putting

$$s = 4E_{0F}^2, \quad (40)$$

with E_{0F} being the relativistic value of the nucleon energy on the Fermi surface. This insures that the nucleon pole on the RHS of Eq. (37) describes the nucleon, added to the Fermi surface. For the analysis, carried out in this section we can neglect the bound, thus putting

$$s = 4m^2. \quad (41)$$

This was the choice of variables in our papers [9]–[11], [32, 33] and [35]–[40]. It was used also in [58], where the approach was used for calculation of the nucleon–nucleus scattering amplitude.

Note that the vacuum dispersion relation (5) can be viewed as a relativistic generalization of the nonrelativistic dispersion relation in time component (energy) q_0 , known as the Lehmann representation [59]. The latter is based on causality. It converts into a dispersion relation in q^2 after being combined with the symmetric relation in negative values of q_0 . The reasoning does not work in medium, since the Lorentz invariance is lost. To prove the dispersion relation (37) we must be sure of the possibility of the contour integration in the complex q^2 plane. A strong argument in support of this possibility is the analytical continuation from the region of real $q^2 \rightarrow -\infty$. At these values the asymptotic freedom of QCD enables one to find an explicit expression for the integrand. The integral over the large circle may have a nonvanishing contribution. However, the latter contains only polynomials in q^2 which are killed by the Borel transform. Thus we consider dispersion relations in q^2 to be a reasonable choice.

On the contrary, dispersion relations in q_0 contain all possible excited states of the matter on its RHS. To illustrate the latter point, consider photon propagation in medium (see, i.e. [60]).

In vacuum the propagator of the photon which carries the energy ω and linear momentum \mathbf{k} is $D_0 = (\omega^2 - k^2)^{-1}$. It has a pole at $q^2 = \omega^2 - k^2 = 0$. In medium it takes the form $D_m = (\omega^2 \varepsilon(\omega, k) - k^2)^{-1}$. Here $\varepsilon(\omega, k)$ is the dielectric function, related to the amplitude of the photon scattering on the ingredients of the medium. If the photon energies are small enough, dependence of ε on k can be neglected (this is known as the dipole approximation). However, $\varepsilon(\omega, 0)$ is a complicated function of ω . It depends on the eigen energies of the medium. The same refers to the function $D_m(\omega)$. However, the function $D_m(q^2)$ still has a simple pole at the point $q^2 = q_m^2 = \omega^2(1 - \varepsilon(\omega))$, reflecting properties of the in-medium photon. Straightforward calculation of the value q_m^2 is a complicated problem. The same is true for the in-medium proton. The SR are expected to provide the in-medium position of the pole in some indirect way.

The approach, in which the dispersion relation in q_0 at fixed value of the three-dimensional momentum $|\mathbf{q}|$ is the departure point was developed in [61]. It was used for description of nucleons [61, 62], delta-isobars [63] and hyperons [64] in nuclear matter. The results are reviewed in [65]. However, the possibility to separate the singularities connected with the nucleon in the matter from those connected with the matter itself in this approach looks to us to be obscure.

Thus we expect the Borel transformed dispersion relations in q^2 with fixed value of s to be more reliable.

3.2 Operator product expansion

Note that the condition $s = \text{const}$ enables to use the OPE of the LHS of Eq. (37). Indeed, we find

$$2(Pq) = s - m^2 - q^2, \quad (42)$$

and thus in the rest frame of the matter

$$q_0 = \frac{s - m^2 - q^2}{2m}.$$

Hence,

$$q^2/q_0 \rightarrow \text{const} = c \sim 2m \quad \text{at} \quad |q^2| \rightarrow \infty. \quad (43)$$

The exponential factor on the RHS of Eq. (35) can be written as

$$e^{i(qx)} = e^{i(q^2 x^2/2cx_z + x_z c/2 + q^2 x_t^2/2cx_z)},$$

with z – the direction of momentum \mathbf{q} , $x^2 = x_0^2 - x_z^2 - x_t^2$. Hence, the integral in Eq. (37) is determined by $x^2 \sim q^{-2}$, $x_z \sim c^{-1}$, and the function $\Xi(x)$ can be expanded in powers of x^2 , corresponding to expansion of $\Pi_m(q^2, s)$ in powers of q^{-2} . Note that condition (43) is the same as that for the validity of the OPE for the structure functions of the deep inelastic scattering [19].

In the case of vacuum the expansion of quark fields $q(x)$ in powers of x was indeed an expansion in powers of x^2 , corresponding to a power series of q^{-2} for the vacuum function $\Pi_0(q^2)$. In medium the fields $q(x)$ can be expanded also in powers of (Px) . This leads to expansion in powers of $(Pq)/q^2$ of the function $\Pi_m(q^2, s)$, and thus, generally speaking, to infinite number of condensates in each OPE term. Fortunately, due to the logarithmic q^2 dependence of the quark loops, the leading OPE terms contain only finite number of condensates. We shall give details in the next Section.

3.3 Model of the spectrum

Now we turn to the RHS of Eq. (37). The function $\Xi(x)$ defined by Eq. (35) can be written as

$$\begin{aligned}\Xi(x) = & \langle M | T j(x) \bar{j}(0) | M \rangle = \langle M_A | j(x) | M_{A+1} \rangle \langle M_{A+1} | \bar{j}(0) | M_A \rangle \theta(x_0) \\ & - \langle M_A | \bar{j}(0) | M_{A-1} \rangle \langle M_{A-1} | j(x) | M_A \rangle \theta(-x_0),\end{aligned}\quad (44)$$

with $|M_K\rangle$ standing for the system with the baryon number K . Here $|M\rangle = |M_A\rangle$ is the ground state, summation over all states with $K = A \pm 1$ is assumed. This equation is illustrated by Fig. 3

The matrix element $\langle M_{A+1} | \bar{j} | M_A \rangle$ contains the term $\langle N | \bar{j} | 0 \rangle$, which adds the nucleon (the “probe nucleon”) to the Fermi surface of the state $|M_A\rangle$, while the rest A nucleons are spectators. If interactions of this nucleon with the matter are neglected, this contribution to $\Xi(x)$ has a pole at $q^2 = m^2$. Interactions of the probe nucleon with the matter shift the position of the pole. In the mean field approximation shown in Fig. 4a the shift does not depend on s . Going beyond the mean field approximation we find the Hartree self-energy diagrams (Fig. 4b), which have singularities in s . Due to condition (41) they do not add new singularities in variable q^2 to the function $\Pi_m(q^2, s)$. The exchange (Fock) self-energy diagram is shown in Fig. 4c.

The matrix element $\langle M_{A+1} | \bar{j} | M_A \rangle$ contains also the terms $\langle B_1 | \bar{j} | 0 \rangle$ with B_1 standing for the system, containing the nucleon and mesons. The current $j(x)$ creates the nucleon and a meson with the mass m_x , which is absorbed by the nucleons of the matter – see Fig. 5. This contribution has a cut in q^2 complex plane, at $q^2 > m^2 + 2m_m m_x$. It has also a cut in s , which is not important for us, since s is fixed.

The matrix element $\langle M_{A-1} | j(x) | M_A \rangle$ contains the terms $\langle B_0 | j(0) | N \rangle$, with B_0 describing a system with the baryon quantum number equal to zero. The other $A-1$ nucleons are spectators. The system B_0 can be a set of π mesons, ω meson, etc. These contributions depend on the variable $u(q^2) = (P-q)^2$, providing singularities at $u > m_x^2$, with m_x the mass of state B_0 – see Fig. 6. Thus the lowest singularity in u corresponds to the branching point $q^2 = m_m^2 + 2m_\pi^2$, corresponding to the real two-pion state in the u channel. A single-particle meson state with the mass m_x generates a pole at $q^2 = m^2 + m_x^2/2$. The diagram, shown in Fig. 4c. is also one of the contributions, which has singularities in the u channel.

Note that the antinucleon state corresponding to $q_0 = -m$ generates the pole $q^2 = 5m^2$, shifted far to the right from the lowest lying state.

Thus the spectrum of the function $\Pi_m(q^2, s)$ consists of the pole at $q^2 = m^2$, a set of higher lying poles, generated by the u channel and a set of branching points. The lowest lying branching point is separated from the position of the pole $q^2 = m_m^2$ by a much smaller distance than in the case of vacuum ($q^2 = m^2 + 2mm_\pi$ in the latter case). Note, however, that at the very threshold the contribution is quenched since the vertices contain linear moments of intermediate pions. Thus the higher singularities can be considered as separated from the pole $q^2 = m_m^2$.

The situation becomes more complicated if we include the interaction of the probe nucleon with $n > 1$ nucleons of the matter. The corresponding amplitudes depend on the variables $s_n = (nP + q)^2$. This causes the cuts, running to the left from the point $q^2 = m^2$. Its contribution thus is not quenched by the Borel transform. However, as we shall see in Sec. 8, such multinucleon interactions require inclusion of the condensates of high dimensions on the

LHS of (37). Such contributions do not have logarithmic loops, thus contributing only to the pole terms on the RHS of Eq. (37). Hence, these singularities should be disregarded in our approach, and the three-body forces are included in the mean-field approximation in our approach.

To summarize the results of this Section, we use Borel transformed dispersion relations of the function $\Pi_m(q^2, s)$ at fixed s . The OPE is used on the LHS. The "pole+ continuum" model is used for the RHS. Now we have three SR equations

$$\begin{aligned} \tilde{\Pi}_m^{i\text{ OPE}}(M^2, s) &= \lambda_{Nm}^2 \xi^i e^{(-m_m^2/M^2)} \\ &+ \frac{1}{2\pi i} \int_{W_m^2}^{\infty} dk^2 \Delta_{k^2} \Pi_m^{i\text{ OPE}}(k^2, s) e^{(-k^2/M^2)}; \quad i = q, P, I. \end{aligned} \quad (45)$$

Here m_m and λ_{Nm}^2 are the position of the nucleon pole in medium and the value of its residue, W_m^2 is the in-medium value of the effective threshold. The meaning of the parameters ξ^i will be clarified in next Section.

4 Nucleon self-energies in the lowest orders of OPE

In the Subsections 4.1 – 4.4 we consider the symmetric nuclear matter, with equal number of protons and neutrons. Also, due to isotope invariance

$$\langle M | \bar{d}d | M \rangle = \langle M | \bar{u}u | M \rangle = \langle M | \bar{q}q | M \rangle. \quad (46)$$

4.1 General equations

Start with description of the nucleon pole. We can write for the inverse nucleon propagator in medium

$$G_N^{-1} = (G_N^0)^{-1} - \Sigma \quad (47)$$

with $G_N^0 = (\hat{q} - m)^{-1}$ being the propagator of the free nucleon, while

$$\Sigma = \hat{q}\Sigma_q + \frac{\hat{P}}{m}\Sigma_P + \Sigma_I \quad (48)$$

is the general expression for the self-energy in the nuclear matter. In the kinematics, determined by Eq. (41) we find

$$G_N = Z \frac{\hat{q} - \hat{P}(\Sigma_V/m) + m^*}{q^2 - m_m^2} \quad (49)$$

with

$$\Sigma_V = \frac{\Sigma_P}{1 - \Sigma_q}; \quad m^* = \frac{m + \Sigma_I}{1 - \Sigma_q}. \quad (50)$$

For the new position of the nucleon pole we find

$$m_m^2 = \frac{(s - m^2)\Sigma_V/m - \Sigma_V^2 + m^{*2}}{1 + \Sigma_V/m}, \quad (51)$$

while

$$Z = \frac{1}{(1 - \Sigma_q)(1 + \Sigma_V/m)}. \quad (52)$$

Thus in Eq. (45)

$$\xi^q = 1; \quad \xi^P = -\Sigma_V; \quad \xi^I = m^*. \quad (53)$$

Note that Eqs. (50) correspond to those for the vector self-energy and for the effective mass in nuclear physics – see, e.g. [66]. Also in accordance with definition accepted in nuclear physics the scalar self-energy is

$$\Sigma_s = m^* - m. \quad (54)$$

In the nonrelativistic limit the proton dynamics is determined by the potential energy

$$U = \Sigma_V + \Sigma_s. \quad (55)$$

Keeping the only terms, which are linear in the density ρ we find a simple expression for the shift of the position of the nucleon pole

$$m_m - m = U. \quad (56)$$

Instead of Eqs. (19) we have now

$$\mathcal{L}_m^i(M^2, W_m^2) = R^i(M^2); \quad i = q, P, I. \quad (57)$$

Here \mathcal{L}_m^i are the Borel transforms of the LHS of Eq. (37), multiplied by $32\pi^4$ – see the previous Section, while

$$\begin{aligned} R^q(M^2) &= \lambda_m^2 e^{-m_m^2/M^2}; \quad R^P(M^2) = -\Sigma_V \lambda_m^2 e^{-m_m^2/M^2}; \\ R^I(M^2) &= m^* \lambda_m^2 e^{-m_m^2/M^2}, \end{aligned} \quad (58)$$

where λ_m^2 is the effective value of the in-medium value of the residue, i.e. $\lambda_m^2 = Z \cdot 32\pi^4 \lambda_{Nm}^2$, following Eqs. (49) and (52).

Employing Eqs. (57) and (58) one finds

$$-\frac{\mathcal{L}_m^P(M^2, W_m^2)}{\mathcal{L}_m^q(M^2, W_m^2)} = \Sigma_V; \quad \frac{\mathcal{L}_m^I(M^2, W_m^2)}{\mathcal{L}_m^q(M^2, W_m^2)} = m^*. \quad (59)$$

4.2 Left-hand sides of the sum rules

4.2.1 Contribution of the condensates of lowest dimension

The calculation is based on the presentation of the single-quark propagator in nuclear matter

$$\begin{aligned} \langle M | T[q_i^a(x) \bar{q}_j^b(0)] | M \rangle &= \frac{i}{2\pi^2} \frac{(\hat{x} - im_q x^2/2)_{ij} \delta^{ab}}{x^4} \\ &- \frac{1}{12} \sum_A \Gamma_{ij}^A \delta^{ab} \langle M | \bar{q}(0) \Gamma^A q(x) | M \rangle, \end{aligned} \quad (60)$$

just analogous to Eq. (14). The bilocal operators in the last term on the RHS of Eq. (60) are not gauge invariant. The gauge invariant expression

$$q(x) = \left(1 + x_\alpha D_\alpha + \frac{1}{2} x_\alpha x_\beta D_\alpha D_\beta + \dots\right) q(0), \quad (61)$$

with D_α standing for the covariant derivatives, provides the infinite series of the local condensates. Anyway, here we are looking for the condensates of the lowest dimension. Hence, we put $q(x) = q(0)$ in the last term of Eq. (60). Thus

$$\begin{aligned} \langle M | T[q_i^a(x) \bar{q}_j^b(0)] | M \rangle &= \frac{i}{2\pi^2} \frac{(\hat{x} - im_q x^2/2)_{ij}}{x^4} \delta^{ab} - \\ &- \frac{1}{12} \delta_{ij} \delta^{ab} \langle M | \bar{q}(0) q(0) | M \rangle - \frac{1}{12} \gamma_{ij}^\mu \delta^{ab} \langle M | \bar{q}(0) \gamma_\mu q(0) | M \rangle. \end{aligned} \quad (62)$$

The first term presents the singular part of the propagator. It is just the same as in vacuum-Eq.(14). We include it in the chiral limit $m_q = 0$. Recall that the calculations in vacuum have been carried out in the chiral limit as well. The second term also has the same form as in vacuum. However, it includes the operator $\bar{q}q$ averaged over the ground state of the matter. The last term contains a new vector condensate, which vanishes in vacuum. In the rest frame of the matter it is

$$v_\mu^q = \frac{1}{2} (v_p^q + v_n^q) \rho \delta_{\mu 0}; \quad v_N^q = \langle N | \bar{q}(0) \gamma_0 q(0) | N \rangle. \quad (63)$$

In the leading order of the OPE two of the quarks are described by the free propagators g_q , while the rest one is presented by one of the two last terms on the RHS of Eq. (62). Present, similar to the vacuum case

$$\Pi_m^q(q^2, s) = \sum_n A_n; \quad \Pi_m^I(q^2, s) = \sum_n B_n; \quad \Pi_m^P(q^2, s) = \sum_n P_n \quad (64)$$

with n denoting the dimension of the condensates. We find immediately that the contribution to Π_m^I can be expressed by Eq.(17) for B_3 with the matrix element $\langle 0 | \bar{d}d | 0 \rangle$ replaced by $\langle M | \bar{d}d | M \rangle$. Thus

$$B_3 = \frac{\langle M | \bar{d}d | M \rangle q^2 \ln(-q^2/L_q^2)}{4\pi^2}. \quad (65)$$

The leading contributions to Π^q and Π^P are determined by the vector condensate

$$\Pi_\rho = \frac{4i}{\pi^4} \int \frac{d^4x}{x^8} \left(\frac{x^2}{2} (\hat{v}^u + \hat{v}^d) + \hat{x}(x, v^u + v^d) \exp(i(q \cdot x)) \right).$$

Direct calculation provides

$$A_{3\rho} = \frac{1}{6\pi^2} \frac{(P \cdot q)}{mL^{4/9}} \ln(-q^2/L_q^2) v(\rho); \quad P_{3\rho} = \frac{q^2}{3\pi^2} \frac{\ln(-q^2/L_q^2)}{L^{4/9}} v(\rho), \quad (66)$$

where $v(\rho) = 3\rho$ is the vector condensate – see Eq. (3). The contributions of these terms to the LHS of Eqs. (57) are

$$\tilde{A}_{3\rho} = -\frac{8\pi^2}{3} \frac{(s - m^2) M^2 E_0(M^2, W_m^2) - M^4 E_1(M^2, W_m^2)}{mL^{4/9}} v(\rho); \quad (67)$$

$$\tilde{B}_{3\rho} = -4\pi^2 M^4 E_1(M^2, W_m^2) \kappa_\rho(\rho); \quad \tilde{P}_{3\rho} = -\frac{8\pi^2}{3} \frac{4M^4 E_1(M^2, W_m^2)}{L^{4/9}} v(\rho).$$

The functions E_n are defined by Eq. (23). Following our notations

$$\kappa_\rho(\rho) = \kappa(\rho) - \kappa(0) \quad (68)$$

We illustrate Eq. (67) by Fig. 7. It describes the exchange by noninteracting quarks with the quarks of the matter. Of course, there are strong interactions between the quarks of the condensate. The corresponding contributions to the nucleon pole (the RHS of Eq. (57)) account for the exchanges by the systems of strongly correlated quarks (mesons) with the same quantum numbers. They have standard Lorentz structures, i.e. the terms proportional to $v(\rho)$ and $\kappa(\rho)$ contribute to vector and scalar structures of Eq. (1) correspondingly. This is illustrated by Fig. 8.

4.2.2 Next to leading OPE terms

Now we include the contribution of the condensates of dimension $d = 4$. These are the contributions caused by nonlocality of the vector condensate $\langle M | \bar{q}(0) \gamma_0 q(x) | M \rangle$ – see Eq. (60) with $q(x)$ defined by Eq. (61) to q and P structures on the LHS of Eqs. (57) and also the contribution of the gluon condensate to the q structure.

Recall that in the case of vacuum there was a contribution A_4 caused by the vacuum gluon condensate – see Eqs. (22) and (24). In the nuclear matter the vacuum gluon condensate should be replaced by its in-medium value. The corresponding contribution is

$$\tilde{A}_{4g\rho} = \pi^2 \frac{M^2 E_0(M^2, W_m^2)}{L^{4/9}} g_\rho(\rho). \quad (69)$$

Here we denoted

$$g(\rho) = \langle M | \frac{\alpha_s}{\pi} G^{a\mu\nu} G_{\mu\nu}^a | M \rangle = g(0) + g_\rho(\rho). \quad (70)$$

Now we consider the contribution of the nonlocal vector condensate. For the sake of simplicity we shall present the results in terms of the nucleon matrix elements, assuming thus that the matrix element $\langle M | \bar{q}(0) \gamma_0 q(x) | M \rangle$ is a linear function of ρ . We can write

$$\theta_\mu^f(x) = \langle M | \bar{q}(0) \gamma_0 q(x) | M \rangle = \frac{P_\mu}{m} \Phi_a^f((Px), x^2) + ix_\mu m \Phi_b^f((Px), x^2). \quad (71)$$

Presenting [67]

$$\Phi_{a,b}^f((Px), x^2) = \int_0^1 d\alpha e^{-i\alpha(P \cdot x)} f_{a,b}^f(\alpha, x^2), \quad (72)$$

we can expand $f_{a,b}^q(\alpha, x^2) = \eta_{a,b}^q(\alpha) + x^2 m^2 \xi_{a,b}^q(\alpha)/8 + O(x^4)$. Here $\eta_a^f(\alpha) = f_a^f(\alpha, 0)$ is the contribution of the valence quark with flavor f to the asymptotics of the nucleon structure function $\eta_a = \eta_a^u + \eta_a^d$, normalized by the condition

$$\int_0^1 d\alpha \eta_a(\alpha) = 3. \quad (73)$$

The moments of the function η_b can be presented in terms of those of η_a and ξ_a [36], e.g.

$$\int_0^1 d\alpha \eta_b(\alpha) = \frac{1}{4} \int_0^1 d\alpha \alpha \eta_a(\alpha). \quad (74)$$

In the equations, displayed in this Subsection we omit some terms, which are not important numerically.

Direct calculation provides the contribution of the nonlocal condensate to \mathcal{L}^P

$$P_{4\rho} = P_{4\rho}^{(1)} + P_{4\rho}^{(2)}; \quad P_{4\rho}^{(1)} = -\frac{1}{6\pi^2 m} \ln \frac{Q^2}{L_p^2} \int_0^1 d\alpha [(5\alpha(P \cdot q') + 2\alpha^2 m^2) \eta_a(\alpha) + 9m^2 \eta_b(\alpha)] \rho;$$

$$P_{4\rho}^{(2)} = \frac{1}{6\pi^2 m} \int_0^1 d\alpha \ln \frac{Q'^2}{Q^2} [(-\alpha(Pq') + 2q'^2) \eta_a(\alpha) - 9m^2 \alpha \eta_b(\alpha)] \rho, \quad (75)$$

with $q' = q - P\alpha$, $Q'^2 = -q'^2$.

To evaluate the contribution $P_{4\rho}^{(2)}$ we rearrange the logarithmic term

$$\ln \frac{Q'^2}{Q^2} = \ln \frac{(1+\alpha)(Q^2 + X^2(\alpha))}{Q^2},$$

with

$$X^2(\alpha) = \frac{\alpha(s - m^2(1+\alpha))}{1+\alpha}.$$

Performing the Borel transform and using the numerical values of the moments of the structure function η_a (actually we used those, presented in [68]) we find all the coefficients of the M^{-2} expansion to be of the same order of magnitude. Hence, following [36] we do not use this expansion here. We employ the explicit expression [36], providing for the contribution to the LHS of Eq. (57)

$$\begin{aligned} \tilde{P}_{4\rho}^{(2)} &= \frac{8\pi^2}{3} \int_0^1 d\alpha \left[(-5(s - m^2)\alpha + 6m^2\alpha^2) G_0(M^2, \alpha) \right. \\ &\quad \left. + (4 + 5\alpha) G_1(M^2, \alpha) \eta_a(\alpha) - 18m^2 \alpha \eta_b(\alpha) \right] \rho. \end{aligned} \quad (76)$$

Here $G_n(M^2, \alpha) = M^{2(n+1)} E_n(X^2(\alpha))$. We included only the lowest moments of the function $\eta_b(\alpha)$, which are related to the moments of the structure function η_a by expression, analogous to Eq. (74) – see [36]. The values of these moments enable to expect the convergence of the latter expansion. It will be useful (see next Subsection) to present the contribution in terms of those of lowest nonvanishing (second) moments of the structure functions. For the P structure it is

$$(\tilde{P}_{4\rho})_2 = \frac{8\pi^2}{3} 5 \frac{(s - m^2) M^2 E_0(M^2, W_m^2) - M^4 E_1(M^2, W_m^2)}{m} \mathcal{M}_{2\rho}, \quad (77)$$

where

$$\mathcal{M}_2 = \int_0^1 d\alpha \alpha \eta_a(\alpha). \quad (78)$$

The conventional notation is $\mathcal{M}_n = \int_0^1 d\alpha \alpha^{n-1} \eta_a(\alpha)$. Numerically $\mathcal{M}_2 = 0.32$ [37].

To avoid the cumbersome formula, we do not present the expressions for the contribution of the nonlocal vector condensate to the q structure of Eq.(57), addressing the readers to our papers [32] and [41]. The contribution of the second moment is

$$(\tilde{A}_{4\rho})_2 = \frac{16\pi^2}{3} m M^2 E_0(M^2, W_m^2) \mathcal{M}_2 \rho. \quad (79)$$

Note that the nonlocality of the scalar condensate vanishes in the chiral limit [36], due to equations of motion. Thus $\tilde{B}_{4\rho} = 0$.

The contribution of the condensate $\langle M | \bar{q} \lambda_{\mu, nu}^a G^{a\mu\nu} \sigma_{\mu\nu} q | M \rangle$ of dimension $d = 5$ vanishes in medium [26], as well as in vacuum [21] if we are using $j = j_1$ – see Eq. (13). There is no such cancellation for the currents j_2 and j_3 . This is one more argument in favor of using the current j_1 , since the in-medium value of this condensate is not known.

4.3 Approximate solution

In this subsection we find an approximate solution of Eq. (57). The nucleon self-energies are expressed explicitly as functions of in-medium QCD condensates.

This solution can be obtained by replacing the in-medium threshold value W_m^2 by its vacuum value W^2 in Eqs. (57). This can be done, since the variation of the threshold δW^2 contains a small factor e^{-W^2/M^2} . The accuracy of the approximate solution will be discussed in the next Subsection.

We put $W_m^2 = W^2$ on the LHS of Eqs. (59) and multiply both numerators and denominators by e^{m^2/M^2} . Taking into account only the terms of dimension $d = 3$ in the medium contributions \mathcal{L}_ρ^i , we can write these equations as

$$\begin{aligned} \frac{\frac{32\pi^2}{3} f_P(M^2) v(\rho)}{\mathcal{L}_0^q(M^2) e^{m^2/M^2} - \frac{8\pi^2}{3m} f_q(M^2) v(\rho)} &= \Sigma_V; \\ \frac{\mathcal{L}_0^I(M^2) e^{m^2/M^2} - 4\pi^2 f_I(M^2) \kappa_\rho(\rho)}{\mathcal{L}_0^q(M^2) e^{m^2/M^2} - \frac{8\pi^2 m}{3} f_q(M^2) v(\rho)} &= m^*. \end{aligned} \quad (80)$$

Here the M^2 dependence of the contributions, provided by medium is contained in the functions

$$\begin{aligned} f_q(M^2) &= \frac{[(s - m^2) M^2 E_0(M^2) - M^4 E_1(M^2)] e^{m^2/M^2}}{L^{4/9}}; & f_P(M^2) &= \frac{M^4 E_1(M^2) e^{m^2/M^2}}{L^{4/9}}; \\ f_I(M^2) &= M^4 E_1(M^2) e^{m^2/M^2} \end{aligned} \quad (81)$$

One can see that they exhibit only weak dependence on M^2 in the interval (27) – see Fig. 9. They can be approximated as

$$\overline{f_q(M^2)} \approx c_0 = 3.29; \quad \overline{f_P(M^2)} \approx c_P = 1.30; \quad \overline{f_I(M^2)} \approx c_I = 1.60.$$

Here the overline means that the function of M^2 is replaced by a constant value, corresponding to the lowest value of the function χ^2 defined by Eq. (26) in the interval (27) at the vacuum

value of the continuum threshold W^2 . The RHS of these equations are written in powers of GeV (their dimensions are GeV^4).

Now one can replace the functions $f_i(M^2)$ by the parameters c_i in Eqs. (80). Also, due to Eqs. (20) and (21) one can replace the functions $\mathcal{L}_0^q(M^2, W^2)e^{m^2/M^2}$ and $\mathcal{L}_0^I(M^2, W^2)e^{m^2/M^2}$ by the constant values λ^2 and $m\lambda^2$ correspondingly. We define also for the contributions induced by the matter

$$\mathcal{F}^i = \frac{\overline{\mathcal{L}_\rho^i(M^2, W^2)}e^{m^2/M^2}}{\lambda^2}.$$

Here the overline has the same meaning as in Eq. (81).

Now Eqs. (59) can be written as

$$\Sigma_V = \frac{\mathcal{F}^P}{1 + \mathcal{F}^q}; \quad m^* = \frac{m + \mathcal{F}^I}{1 + \mathcal{F}^q}. \quad (82)$$

This is a direct analog of Eq. (50). We can identify

$$\Sigma_q = -\mathcal{F}^q. \quad (83)$$

We denote also

$$\mathcal{F}^i = \sum_n \mathcal{F}_n^i, \quad (84)$$

where \mathcal{F}_n^i is the contribution of the condensates with the dimension $d = n$.

Employing Eq. (68) we find the contribution of the condensates with $d = 3$. In Eqs. (82) and (83)

$$\mathcal{F}_3^q = -47v(\rho); \quad \mathcal{F}_3^P = 66v(\rho); \quad \mathcal{F}_3^I = -32\kappa_\rho(\rho), \quad (85)$$

with all values written in powers of GeV. Using Eq. (3) we immediately find the values of Σ_q and Σ_V at any value of the density ρ . For example, at phenomenological value of saturation density

$$\rho = \rho_0 = 0.16 \text{ Fm}^{-3} = 1.3 \cdot 10^{-3} \text{ GeV}^3 \quad (86)$$

we obtain

$$\Sigma_V = 314 \text{ MeV}, \quad \Sigma_q = 0.18. \quad (87)$$

Now we include the condensates of dimension $d = 4$. The contribution of the gluon condensate can be obtained by employing Eq. (69) at $W_m^2 = W^2$. We present the contributions of the nonlocal vector condensate at $W_m^2 = W^2$ through the contributions \mathcal{F}_4^{iL} , corresponding to inclusion of only the second moments of the structure functions \mathcal{M}_2 , given by Eqs. (77) and (79). Direct calculation provides $\mathcal{F}_{4\rho}^{PL} = 290\mathcal{M}_2\rho$, and $\mathcal{F}_{4\rho}^{qL} = 55\mathcal{M}_2\rho$ in GeV units, while

$$\mathcal{F}_4^P = 0.37\mathcal{F}_4^{PL}; \quad \mathcal{F}_4^q = 0.62\mathcal{F}_4^{qL}. \quad (88)$$

Thus, with inclusion of the condensates of dimension $d = 3, 4$

$$\mathcal{F}_q = -47v(\rho) + 8.6g_\rho + 34\mathcal{M}_2\rho. \quad (89)$$

Also, from Eq. (78) we find

$$\mathcal{F}_P = 66v(\rho) - 106\mathcal{M}_2\rho, \quad (90)$$

while

$$\mathcal{F}_I = -32\kappa_\rho(\rho), \quad (91)$$

with the RHS written in GeV units. The value of \mathcal{F}^I remained unchanged. Recall that $\mathcal{M}_2 = \int_0^1 d\alpha \alpha \eta_a(\alpha) = 0.32$ is the second moment of the nucleon deep inelastic structure function.

4.4 Gas approximation

In the lowest order of OPE expansion the nucleon scalar self-energy depends on the scalar condensate $\kappa(\rho)$ defined by Eq. (4). The next to leading order terms contain the gluon condensate $g(\rho)$, defined by Eq. (70). Below we shall consider the problem of finding the density dependence of these condensates. In this Subsection we include them in the framework of the gas approximation [24, 25]

$$\kappa(\rho) = \kappa(0) + \kappa_N \rho; \quad \kappa_N = \langle N | \sum_i \bar{q}^i q^i | N \rangle, \quad (92)$$

and

$$g(\rho) = g(0) + g_N \rho; \quad g_N = \langle N | \frac{\alpha_s}{\pi} G^{a\mu\nu} G_{\mu\nu}^a | N \rangle. \quad (93)$$

Thus in this approximation $\kappa_\rho = \kappa_N \rho$, $g_\rho = g_N \rho$.

The scalar quark condensate is numerically more important. Numerous model calculations – see, e.g. [14, 69], and [70] for earlier papers, provide relatively small nonlinear term $S(\rho)$ on the RHS of Eq. (4) at ρ close to the saturation value ρ_0 . Thus, employing of the gas approximation is reasonable.

Now we must find the values of κ_N and g_N .

4.4.1 The value of $\langle N | \bar{u}u + \bar{d}d | N \rangle$

The expectation value κ_N is connected to the pion-nucleon sigma-term σ by the relation [28]

$$\kappa_N = \langle N | \bar{u}u + \bar{d}d | N \rangle = \frac{2\sigma}{m_u + m_d}. \quad (94)$$

On the other hand, the σ term can be expressed in terms of the pion–nucleon elastic scattering amplitude $T = T(s, t, k^2, k'^2)$ [28]. Here k and k' are the pion momenta before and after scattering, s and t are the Mandelstam variables $s = (p + k)^2$ and $t = (k' - k)^2$, with p denoting the momentum of the nucleon. The σ term σ is connected to the amplitude F in the unphysical point

$$T(0, 0, 0, 0) = -\frac{\sigma}{f_\pi^2}, \quad (95)$$

where f_π is the pion decay constant. The experiments provide the data on the physical amplitude $T_{phys} = T((m + m_\pi)^2, m_\pi^2, m_\pi^2, m_\pi^2)$. The method of extrapolation of observable physical amplitude to the unphysical point was worked out in [29]. The Σ term $\Sigma^{\pi N} = -T_{phys} f_\pi^2$ differs from the σ term by

$$\Sigma^{\pi N} - \sigma = 15 \text{ MeV}. \quad (96)$$

Note that $(\Sigma^{\pi N} - \sigma)/\sigma \sim m_\pi$, in the chiral limit we must put $\Sigma = \sigma$.

For many years the value

$$\sigma = (45 \pm 8) \text{ MeV}, \quad (97)$$

based on the results of [71] was assumed to be true [72]. This corresponds to $\Sigma^{\pi N} \approx 60 \text{ MeV}$. However, some of the latest results support rather the value of $\Sigma^{\pi N}$ close to 80 MeV [73, 74, 75], corresponding rather to

$$\sigma = (60 - 70) \text{ MeV}. \quad (98)$$

This is somewhat closer to some of results, claimed earlier [76].

There is large discrepancy between the theoretical results – see [70] for references. Note that some of the latest ones [77, 78] support the value $\sigma = 45 \text{ MeV}$.

4.4.2 The value of $\langle N | \frac{\alpha_s}{\pi} G^{a\mu\nu} G_{\mu\nu}^a | N \rangle$

This expectation value was calculated in [79] by averaging the trace of the QCD energy-momentum tensor Θ over the nucleon state. The trace is

$$\Theta_\mu^\mu = \sum_i m_i \bar{q} q_i - \frac{b\alpha_s}{8\pi} G^2, \quad (99)$$

where m_i the mass of the quark with the flavor i , $G^2 = G^{a\mu\nu} G_{\mu\nu}^a$, $b = 11 - 2n/3$, n is the total number of flavors. There is a remarkable cancellation [79]

$$\langle N | \sum_j m_j \bar{q}_j q_j | N \rangle - \frac{2n_h}{3} \frac{1}{8} \langle N | \frac{\alpha_s}{\pi} G^2 | N \rangle = 0. \quad (100)$$

Here j stand for the “heavy” quarks with the masses m_j much larger than the inverse confinement radius μ (the accuracy of Eq. (100) is $(\mu/m_j)^2$); n_h is the number of heavy quarks. Hence, the charmed quark is the lightest one among those, which contribute to the LHS of Eq. (100), and

$$g_N = -\frac{8}{9} (m - \sum_i m_i \langle N | \sum \bar{q}^i q^i | N \rangle), \quad (101)$$

with $i = u, d, s$. The simplest expression for g_N can be obtained in the chiral $SU(3)$ limit

$$g_N = g_N^{(1)} = -\frac{8}{9} m. \quad (102)$$

One can see that

$$0 > g_N > g_N^{(1)}. \quad (103)$$

Various estimations for the strange condensate in nucleon $\langle N | \bar{s} s | N \rangle$ provide controversial results (see, e.g. [71]). However, since contribution of the gluon condensate is small even for $g_N = g_N^{(1)}$, the estimation (103) is sufficient for our analysis.

4.4.3 Numerical results in the gas approximation

The results of previous subsection enable us to obtain the complete set of numerical results. We compare the approximate solution given by Eqs. (85), (89), (90), (91) and “exact” solution of the system of Eqs. (57) at the saturation value of density $\rho = \rho_0$.

Start with the value $\kappa_N = 8$, which is consistent with Eq.(97). Including only the condensates of dimension $d = 3$ we find for approximate solution $\Sigma_V = 314$ MeV, $m^* - m = -205$ MeV. Note that due to relatively large value $\Sigma_q = 0.18$ there is a contribution of about +200 MeV to $m^* - m$, which is proportional to the *vector* condensate, and the one of about -400 MeV, proportional to the *scalar* condensate. Solving Eqs. (57) we find

$$\begin{aligned}\Sigma_V &= 323 \text{ MeV}, & m^* - m &= -201 \text{ MeV}, \\ \lambda_m^2 - \lambda^2 &= -0.077 \text{ GeV}^6, & W_m^2 - W^2 &= 0.12 \text{ GeV}^2.\end{aligned}\tag{104}$$

Thus, indeed the approximate solution is a very accurate one.

Turning to the condensates of dimension $d = 4$ and using Eq.(103) we see the gluon condensate provides a small contribution of 0.009 to Σ_q in the approximate solution. Thus it adds about 3 MeV to both Σ_V and m^* . The nonlocal vector condensate subtracts 0.014 from the value of Σ_q . The nonlocality of the vector condensate subtracts also 54 MeV from Σ_V . The approximate solution provides now $\Sigma_V = 257$ MeV, $m^* - m = -202$ MeV. Inclusion of the condensates with dimension $d = 4$ changes the solution of Eq.(57) to

$$\begin{aligned}\Sigma_V &= 253 \text{ MeV}, & m^* - m &= -213 \text{ MeV}, \\ \lambda_m^2 - \lambda^2 &= -0.23 \text{ GeV}^6, & W_m^2 - W^2 &= 0.04 \text{ GeV}^2.\end{aligned}\tag{105}$$

For $\kappa = 11$, corresponding to $\sigma = 60$ MeV (98) we find

$$\Sigma_V = 238 \text{ MeV}, \quad m^* - m = -368 \text{ MeV}.$$

Fixing the continuum threshold $W_m^2 = W^2$ we would obtain

$$\Sigma_V = 276 \text{ MeV}, \quad m^* - m = -336 \text{ MeV}.$$

Thus the approximate solution, discussed above becomes less precise at larger values of κ .

We show the density dependence of the nucleon parameters and of the effective threshold value W_m^2 for $\kappa_N = 8$ in Fig. 10. The approximate solution provided by Eqs. (89)–(91) is shown by dashed line in Fig.10 a. In Fig. 11 we show the dependence of these parameters on κ_N at saturation value of nuclear density ρ_0 .

4.5 Asymmetric matter

4.5.1 Condensates of the lowest dimension

Now we employ the SR approach for calculation of the nucleon self-energies in the nuclear matter, composed of neutrons and protons, distributed with the different densities ρ_n and

ρ_p , i.e. in the $SU(2)$ asymmetric matter. We calculate the dependence on the total density $\rho = \rho_n + \rho_p$ and on asymmetry parameter

$$\beta = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}. \quad (106)$$

We shall calculate the parameters for the proton. Those for the neutron can be obtained by changing β to $-\beta$. As well as in the case of symmetric matter the lowest order OPE terms depend on the vector and scalar condensates. However, in addition to the isospin symmetric condensates $v(\rho)$ and $\kappa(\rho)$, defined by Eqs. (3), (4), we have two $SU(2)$ asymmetric condensates of dimension $d = 3$. These are the vector condensate

$$v_\mu^{(-)}(\rho) = \langle M | \bar{u} \gamma_\mu u - \bar{d} \gamma_\mu d | M \rangle, \quad (107)$$

and a scalar one

$$\zeta_m(\rho, \beta) = \langle M | \bar{u} u - \bar{d} d | M \rangle. \quad (108)$$

The lower index m is introduced in order to show that for the $SU(2)$ invariant vacuum $\zeta_m(0) = 0$.

The vector isotope asymmetric condensate can be immediately calculated. In the rest frame of the matter

$$v_\mu^{(-)}(\rho) = v_0^{(-)}(\rho, \beta) \delta_{0\mu}; \quad v^{(-)}(\rho, \beta) = -\beta \rho. \quad (109)$$

The situation with the scalar condensate is more complicated. Even in the gas approximation, where

$$\zeta_m(\rho, \beta) = -\beta \rho \zeta_p; \quad \zeta_p = \langle p | \bar{u} u - \bar{d} d | p \rangle \quad (110)$$

cannot be directly related to observables. Thus we need some model assumptions.

Considering the nucleon as a system of three valence quarks the sea of the quark-antiquark pairs we can write

$$\zeta_p = \zeta_p^v + \zeta_p^s,$$

where the upper indices denote the contributions of the valence and sea quarks correspondingly. In nonrelativistic quark models $\zeta_p^v = 1$, while $\zeta_p^v < 1$ for the relativistic models. The experimental data on the deep inelastic scattering provide $\zeta_p^s \approx -0.15$ [80, 81].

4.5.2 Nucleon self-energies

In this Subsection we include only the condensates of the lowest dimension $d = 3$. We can write the contributions to the LHS in a form similar to Eqs. (67). The term $\tilde{A}_{3\rho}^q$ depends only on the vector condensate $v(\rho)$, and does not depend on β . Thus it remains the same as for the symmetric matter, while

$$\begin{aligned} \tilde{B}_{3\rho} &= -4\pi^2 M^4 E_1(M^2, W_m^2) \theta(\rho, \beta); \\ \tilde{P}_{3\rho} &= -\frac{8\pi^2}{3} \frac{4M^4 E_1(M^2, W_m^2)}{L^{4/9}} w(\rho, \beta). \end{aligned} \quad (111)$$

The functions E_n are given by Eq. (23). Here we defined $\theta(\rho, \beta) = (\kappa_N + \beta \zeta_N) \rho$ and $w(\rho, \beta) = 3\rho(1 - \beta/4)$. Comparing these expressions with Eq.(67) we find a simple solution in which the proton self energies are expressed in terms of the solution for symmetric matter [33].

$$\Sigma_V(\rho, \beta) = \Sigma_V(\rho, 0)(1 - \beta/4); \quad m^*(\rho, \kappa_N, \beta, \zeta_p) = m^*(\rho, \kappa_N + \beta \zeta_p, 0, 0); \quad (112)$$

$$W_m^2(\rho, \beta) = W_m^2(\rho, 0).$$

For example, employing Eq.(104) we find that in the neutron matter ($\beta = 1$) the proton vector self-energy is 242 MeV, while for neutron it is 403 MeV. We shall see that inclusion of the higher condensates will modify the values.

4.6 Possible mechanism of saturation

4.6.1 A special role of the pion cloud

Note that the simplest account of the nonlinear effects signals on a possible saturation mechanism in our approach [24]–[26]. The nucleons of the matter interact by meson exchange, and the nonlinear contribution to the scalar condensate $S(\rho)$ – see Eq. (4) comes from the meson cloud, created by nucleons. The contributions of each meson is thus proportional to the value of the operator $\bar{q}q$ averaged over the meson states.

We expect exchanges by pions to dominate the value $S(\rho)$. The matrix element $\langle h | \sum \bar{q}^i q^i | h \rangle$ counts the total number $n_{\bar{q}q}$ of quarks and antiquarks in a hadron h [82, 83, 84] under certain reasonable assumptions on the quark wave function of the hadron [83, 84]. Thus one can expect $\langle \mu_j | \bar{q}^i q^i | \mu_j \rangle \sim 2$ for a meson μ . However for the pion, which is a collective Goldstone excitation

$$\langle \pi | \sum \bar{q}^i q^i | \pi \rangle = \frac{2m_\pi^2}{m_u + m_d} = 2m_\pi n_{\bar{q}q}, \quad (113)$$

(the factor $2m_\pi$ comes from normalization of the pion vector of state). Thus $n_{\bar{q}q} \approx 12$ for pions, and we expect their exchanges to provide the leading contribution to $S(\rho)$.

4.6.2 Lowest order in Fermi momentum series

Here we consider the case of symmetric matter. We shall calculate the contribution the non-linear term $S(\rho)$ in the lowest order of expansion in powers of Fermi momentum. The latter is related to the density as

$$\rho = \frac{2}{3\pi^2} p_F^3. \quad (114)$$

The leading contribution is provided by single-pion exchange term (known also as the Fock term or the Pauli blocking term). It can be written as

$$S(\rho) = - \int \frac{d^3 p_1}{(2\pi)^3} \frac{d^3 p_2}{(2\pi)^3} \Gamma(k) D^{(0)2}(k) \Gamma(k) \langle \pi | \bar{u}u + \bar{d}d | \pi \rangle, \quad (115)$$

where $p_{1,2} \leq p_F$ are the nucleon momenta, $\mathbf{k} = \mathbf{p}_1 - \mathbf{p}_2$, Γ stand for the πNN vertices, $D^{(0)} = 1/(\omega^2 - k^2 - m_\pi^2 + i\varepsilon)$ is the propagator of free pion carrying the energy ω . Summation over spin and isospin variables is assumed. The corresponding contribution to the correlator is illustrated by Fig. 12. The function $S(\rho)$ is obtained analytically [36]. The expression becomes especially simple in the chiral limit $m_\pi^2 = 0$

$$S(\rho) = - \frac{9}{32\pi^2} \frac{p_F \rho}{f_\pi^2} \frac{2m_\pi^2}{m_u + m_d}, \quad (116)$$

where the last factor is the pion expectation value Eq. (113). Thus $S(\rho) \sim \rho^{4/3}$. Note that the heavier mesons do not contribute to the leading term of the Fermi momentum expansion. Their contribution is of the order ρ^2 .

The difference between the RHS of Eq. (116) and the result, obtained with the use of the physical value of m_π^2 is rather large. However, in the chiral limit one should use the value of Σ term, rather than that of σ -term for the calculation of κ_N – see Eq. (96), since the difference $\Sigma - \sigma$ contains additional powers of m_π . This diminishes the difference. As a result, the value of κ with κ_N replaced by

$$\kappa_N^{ch} = \frac{2\Sigma}{m_u + m_d} \quad (117)$$

appears to be close to that, obtained for the physical pion mass [30]. Some additional arguments supporting the use of the chiral limit at ρ close to ρ_0 were given in [85].

Now we expand the nucleon potential energy $U(\rho)$, defined by Eq. (55) in powers of Fermi momentum. We take into account the contributions of the order ρ and $\rho p_F \sim \rho^{4/3}$, neglecting, however, the terms of the order ρ^2 . Including only the condensates of the lowest dimension $d = 3$ and using Eq. (85) we find (in GeV units)

$$\Sigma_V = 66v(\rho); \quad \Sigma_s = 46v(\rho) - 32\kappa_\rho(\rho). \quad (118)$$

Here $\kappa_\rho(\rho) = \kappa_N^{ch}\rho + S(\rho)$ with $S(\rho)$ given by Eq. (116). Thus the nucleon energy on the Fermi surface can be written as

$$\mu(z) = [38z^{2/3} + (430 - 42\kappa_N^{ch})z + 126z^{4/3}] \text{ MeV}. \quad (119)$$

Here $z = \rho/\rho_0$, with ρ_0 being the phenomenological value of saturation density – Eq. (86). The first term on the RHS is the kinetic energy.

4.6.3 Equation of state

The saturation value ρ can be found by minimization of the binding energy per particle

$$\epsilon(\rho) = \frac{1}{\rho} \int_0^\rho dx \mu(x). \quad (120)$$

In our case

$$\epsilon(z) = [23z^{2/3} + (215 - 21\kappa_N^{ch})z + 54z^{4/3}] \text{ MeV}. \quad (121)$$

One can see that $\epsilon(z)' = 0$ at $z = 1$ if $\kappa_N^{ch} = 14.4$. This corresponds to $\Sigma^{\pi N} = 79 \text{ MeV}$, in agreement with the experimental data. The nucleon energy at the Fermi surface is $\mu = -11 \text{ MeV}$. The vector self-energy is $\Sigma_v \approx 260 \text{ MeV}$, while $m - m^* \approx -310 \text{ MeV}$. The value of incompressibility $K = 9\rho_0^2 d^2 \epsilon / d\rho^2 = 170 \text{ MeV}$. A standard value obtained in the nuclear physics approaches is $K \approx 230 \text{ MeV}$ [86]. Inclusion of the condensates with $d = 4$ modifies these values slightly.

Note that somewhat similar saturation mechanism take place in the model, suggested later in [87]. The approach of [87] was based on the chiral effective Lagrangian. The authors calculated the nonlinear contribution to the scalar condensate, caused by the pion exchange. The value of $S(\rho)$ appeared to be close to the ours one.

Of course, one should not take these results too seriously. We tried to obtain the values of the self-energies. The calculations of the potential energy require somewhat higher accuracy, since it is a result of subtraction of two large values. Thus, reasonable values for the potential energy is a surprise. Of course, we can not expect accurate values for the binding energy. Also, the results are very sensitive to the exact value of κ_N^{ch} . For example, in this approach the saturation is achieved at $\rho = 2\rho_0$ if $\kappa_N^{ch} = 14.7$, corresponding to $\Sigma = 83$ MeV. Also, the contribution of two-pion exchange to $S(\rho)$ is too large to be neglected [88].

However, the results of this subsection can be treated as a sigh of a possible mechanism for saturation of nuclear matter. It is due to the nonlinear behavior of the scalar condensate. Thus in our approach the saturation is possible only if the three-body interactions (strictly speaking, many-body interactions) are included.

The potentials of the form $U(z) = C_1 z + C_{4/3} z^{4/3}$ were studied earlier in nuclear physics. Such potential with $C_1 \approx -210$ MeV, $C_{3/4} \approx 160$ MeV was analyzed in [89]. Note that our values in Eq.(119) are $C_1 = -175$ MeV (for $\kappa_N^{ch} = 14.4$) and $C_{4/3} = 126$ MeV.

Note that this mechanism differs from that of the Walecka model [1, 2]. Recall that in the latter case the saturation is due to the different behavior of the vector and scalar fields on density. The vector and scalar fields can be expressed as [1]

$$V = \int \frac{d^3 p}{(2\pi)^3} N_{v(s)}(p) g_v \theta(p_F - p); \quad \Phi = \int \frac{d^3 p}{(2\pi)^3} N_{v(s)}(p) g_s \theta(p_F - p) \quad (122)$$

with the sum over spin and isospin variables being assumed. Here g_v and g_s are the coupling constants. In the vector case $N_v = \bar{u}_N(p) \gamma_0 u_N(p) / 2E(p)$ with u_N standing for the nucleon bispinor, while $E(p) = V(\rho) + (m^*(\rho) + p^2)^{1/2}$. One finds immediately that $N_v = 1$, and $V(\rho) = g_v \rho$ is exactly proportional to ρ . In the scalar case $N_s(p) = \bar{u}_N(p) u_N(p) / 2E(p)$, leading to a more complicated dependence $\Phi(\rho_s) = g_s \rho_s(\rho)$, where the scalar density

$$\rho_s = \int \frac{d^3 p}{(2\pi)^3} N_{(s)}(p) g_\theta \theta(p_F - p) \quad (123)$$

is a nonlinear function of ρ . Thus, the saturation in Walecka model is a relativistic effect. It takes place in framework of the two-body interactions. On the contrary, in our approach it does not require relativistic treatment of the nucleons of the matter, being caused by the many-body interactions.

We shall return to this problem in Sec. 9.

5 Other characteristics of the in-medium nucleons

5.1 Axial coupling constant

The SR method have been used for calculation of the nucleon coupling constants for isovector and isoscalar axial currents in vacuum [90]-[92]. The approach is based on considering the function Π_0 defined by Eq. (12) and the dispersion relation (9) in external axial field A_μ , coupled to the quarks by interaction $V = g_q A_\mu \gamma^\mu \gamma_5$, and included in the lowest order of the perturbation theory. In the isovector case, directly related to the neutron β decay $g_u = 1, g_d = -1$. The

corresponding nucleon coupling constant in vacuum is $g_A = 1.27$ [56]. Recall that the vector coupling constant is $g_V = 1$, and the nonzero difference $g_A - g_V$ is due to the nonconservation of the chiral current.

The function Π_0 contains several structures, which are linear in A . The structure $\hat{q}(A \cdot q)\gamma_5$ appeared to be the most convenient one for the SR analysis. The LHS contains now several expectation values, which vanish in the absence of the axial field. The condensate $\iota_\alpha = \langle 0 | \bar{q} \tilde{G}_{\alpha\beta} \gamma_\beta q | 0 \rangle_A = A_\alpha \nu \langle 0 | \bar{q} q | 0 \rangle$, with ν usually referred to as the field induced susceptibility appeared to be numerically the most important one. The numerical value of this expectation value was obtained in [93].

On the RHS of the SR the lowest lying singularity is now the double pole, corresponding to the contribution $\langle 0 | \bar{j} | N \rangle \langle N | \hat{A} \gamma_5 | N \rangle \langle N | \bar{j} | 0 \rangle$. The SR reproduced the experimental value of the isovector coupling constant $g_A \approx 1.25$.

The first attempt to calculate the value of g_A^m for the nucleon in nucleon matter was made in [94]. The authors employed Eq. (30), combining it with several phenomenological assumptions.

In [38] the renormalization of the nucleon coupling constant $\delta g_A = g_A^m - g_A$ in nuclear matter was calculated by extending of the method developed in [24]–[25] for the case of the external axial field. The LHS of the SR was dominated by the configuration, in which one of the $\bar{q}q$ pairs was exchanged with the condensate ι_α , and the other one with the quarks of the nucleons of the matter. The RHS of the SR included the contribution of the isobar-hole (ΔN) excitations in terms of the single-pole terms.

The result of calculation depend on the value of the scalar condensate in nuclear matter. Thus it depends on the actual value of the σ term. It was obtained that $\delta g_A = -0.24$ at the saturation density if $\sigma = 60$ MeV. For $\sigma = 45$ MeV the shift is $\delta g_A = -0.22$. This is in agreement with the experimental data [95, 96], providing $g_A^m(\rho_0) \approx 1.0$. If the single-pole terms are not included, the result is $\delta g_A = -0.05$. Hence, the $\Delta - N$ excitations is the main mechanism of the process, in agreement with [9, 10].

However, the approximations for the LHS of the SR, made in [38] are too crude. A more rigorous analysis, which includes the four-quark condensates is still needed.

5.2 Charge-symmetry breaking forces

5.2.1 Neutron–proton mass difference

It is known [56] that the difference between the neutron and proton masses in vacuum is $m_{np} = 1.3$ MeV, while the electromagnetic interactions contribute to this value is $m_{np}^e \approx -0.7$ MeV [11]. Thus the contributions of the strong interactions is $m_{np}^s \approx 2.0$ MeV. If the isospin symmetry (known also as the charge symmetry) in the hadron interactions is assumed, the value of m_{np}^s would not change in nuclear matter. Thus, it is interesting to study the density dependence $m_{np}^s(\rho)$.

One can expect the mass difference of mirror nuclei to be

$$\Delta M = E^e + m_{np}, \quad (124)$$

where E^e is the electromagnetic energy difference. The values of ΔM and E^e can be, correspondingly, measured and calculated with high accuracy [97]. However, this equation is not

satisfied if m_{np} is equaled to its vacuum value [98]. The discrepancy between the two sides of Eq.(124) increases with the atomic number A , reaching 0.9 KeV for $A = 208$. This is called the Nolen–Schiffer anomaly (NSA).

The phenomenological charge-symmetry breaking (CSB) potentials of the nucleon interactions were discussed in [11]. Some of them described the NSA, but contradicted the data on the CSB effects in NN scattering. A possible resolution of the NSA is the assumption that the value of m_{np} in a nuclei is smaller than that in vacuum. In the case of nuclear matter this assumption would manifest itself in the decrease of $m_{np}(\rho)$ while ρ increases. The renormalization of m_{np} was calculated in numerous papers (see, e.g. [99]–[103]) for nuclear matter and for the finite nuclei. However, the NSA is not fully understood yet.

5.2.2 QCD sum rules view

The QCD SR provide a consistent formalism for the CSB effects. In QCD the charge-symmetry is broken due to a nonzero value of the difference of current quark masses

$$m_d \approx 7 \text{ MeV}; \quad m_u \approx 4 \text{ MeV}; \quad \mu = m_d - m_u \approx 3 \text{ MeV}. \quad (125)$$

Besides the quark mass difference the CSB manifests itself also through a nonzero expectation value of the operator $\bar{u}u - \bar{d}d$. In vacuum the ratio

$$\gamma_0 = \frac{\langle 0 | \bar{d}d - \bar{u}u | 0 \rangle}{\langle 0 | \bar{u}u | 0 \rangle}$$

was obtained by the QCD SR approach [104, 105, 106] basing on the experimental values of isospin breaking mass splitting of nucleons and hyperons.

In nuclear matter one can expect the charge-symmetry breaking effects to be expressed in terms of the quark mass difference μ and of the isospin symmetry breaking condensate

$$\gamma_m = \frac{\langle M | \bar{d}d - \bar{u}u | M \rangle}{\langle M | \bar{u}u | M \rangle}$$

The first attempt to calculate $m_{np}(\rho)$ was carried out in [107]. The analysis was based on the vacuum SR, obtained in [104]. Only the terms, containing scalar condensate were included. The vacuum scalar condensate in equations of [104] was replaced by its in-medium value, obtained under certain phenomenological assumptions. Calculations of [108, 109] included also the density dependence of the CSB parameter γ_m .

The SR calculations of [39] present the direct extension of the approach, developed in [36]. Now we must calculate the contribution of the strong interactions to the difference of the binding energies $\varepsilon_{np} = \varepsilon_n - \varepsilon_p$ of the neutron and proton in symmetric nuclear matter. One can write $\varepsilon_i = U_i + T_i$, with $U_i(T_i)$ - the potential (kinetic) energy of the nucleon $i = p, n$. The potential energies are expressed in terms of the self-energies by Eq. (55), while $T_i = p_F^2/2m_i^*$.

Now the LHS of Eqs. (57) should be calculated for the neutron and proton separately. The explicit dependence of the free quark propagators g_q , defined by Eq. (15) on the quark masses should be taken into account. Also, the term \mathcal{L}_m^I on the LHS of Eq. (57) contains the CSB condensate $\langle M | \bar{u}u - \bar{d}d | M \rangle$. Thus the difference of the binding energies can be written as

$$\varepsilon_{np}(\rho) = \mu b_1(\rho) + \gamma_m(\rho) b_2(\rho). \quad (126)$$

The functions $b_{1,2}(\rho)$ can be obtained by the SR method. The density dependence of the condensate γ_m can be obtained in framework of certain models.

Start with the calculation of $b_{1,2}$. The terms, proportional to the quark mass difference contribute to the difference of the scalar self-energies

$$\delta_1 = 0.17\mu \frac{v(\rho)}{\rho_0}, \quad (127)$$

and to the difference of the vector ones

$$\delta_2 = -0.029\mu \frac{\kappa_\rho(\rho)}{\rho_0}. \quad (128)$$

Recall that $v(\rho) = 3\rho$ is the vector condensate, $\kappa_\rho(\rho) = \kappa(\rho) - \kappa(0)$, with the scalar condensate κ defined by Eq. (4), ρ_0 is the value of saturation density, given by Eq.(86). There is also a contribution to the difference of the scalar self-energies, containing the CSB condensate γ_m

$$\delta_3 = C(\gamma_m(\kappa(\rho) - \kappa(0)) + (\gamma_m - \gamma_0)\kappa(0)); \quad C = 32 \text{ GeV}^{-2}. \quad (129)$$

One should include also the strong-interaction part to the neutron-proton difference of the vacuum parameters m_{np} , λ_{np}^2 and W_{np}^2 . The two last ones were obtained in [105] by the SR method, basing on the empirical value of m_{np}^s . These parameters, as well as the CSB condensate γ_0 were expressed through the quark mass difference μ .

We present the values of $b_{1,2}$ at the saturation value of the density

$$b_1(\rho_0) = -0.73; \quad b_2(\rho_0) = -1.0 \text{ GeV}. \quad (130)$$

The density behavior $\gamma_m(\rho) = \gamma_0(\kappa(\rho)/\kappa(0))^{1/3}$, based on the Nambu–Jona–Lasinio model was suggested in [108]. Under this assumption and employing $\gamma_0 = -2 \cdot 10^{-3}$ [105] we obtain $\varepsilon_{np}(\rho_0) = -0.9 \text{ MeV}$. Our final result is

$$\varepsilon_{np}(\rho_0) = (-0.9 \pm 0.6) \text{ MeV}, \quad (131)$$

with the errors caused mostly by uncertainties of the value of γ_0 .

Thus at least qualitative explanation of the NSA is achieved.

5.2.3 Consequence for conventional nuclear physics

At least two points of the QCD SR analysis can be useful for nuclear physics.

In conventional nuclear physics the $\omega - \rho$ mixing in the vector channel is usually believed to be responsible for the largest part of the Nolen–Schiffer anomaly. However, the SR analysis predicts the scalar channel to be important as well. Neglecting the terms, containing the scalar condensate, we would obtain $\varepsilon_{np}(\rho_0) > 0$, This contradicts the experimental data and general theoretical expectations.

The other point is the Lorentz structure of the nucleon interactions. In the QHD Eq. (1) the terms \hat{V} and Φ are caused by the vector and scalar interactions correspondingly. In the SR analysis described in Sec. 4. these terms are determined by the vector and scalar condensates

correspondingly. Such separation is violated by inclusion of the mass terms in the quark propagators g_q determined by Eq. (15). Relation between the Lorentz structures of Eq. (1) and the condensates becomes more complicated. Now there are contributions to Φ , which are proportional to the vector condensate and contributions to \hat{V} , proportional to the scalar condensate. These admixtures are small, being proportional to the current quark masses. This anomalous Lorentz structure are shown in Fig. 13.

In the case of the CSB forces they manifest themselves in the leading terms. One can see that δ_1 given by Eq. (127) contributes to the structure Φ of Eq. (1), being determined by the vector interaction. On the other hand, the contribution δ_2 presented by Eq. (128) is determined by interaction in the scalar channel, but contributes to the vector structure V . This correspond to anomalous structure of the nucleon vertex functions.

These points can be helpful in constructing the CSB nucleon forces.

5.3 Nucleon deep inelastic structure functions

The SR approach enables to investigate internal structure of the nucleon. Such problems are inaccessible for nuclear physics.

5.3.1 Nucleon structure functions in QCD sum rules

In deep inelastic scattering (DIS) the electrons transfer large energies and momenta to the target. This process is a well known tool for investigation of the internal structure of the latter. The cross section of DIS can be expressed through the imaginary part of the amplitude of the elastic scattering of the virtual photon. The latter has a large and negative four-momentum squared $k^2 < 0$, $-k^2 \gg m^2$ (m is the rest mass of the target) and the energy $k_0 \gg m$, while $-k^2/k_0 \sim m$. The investigation of DIS enables to study the momentum distribution of the quarks. The structure functions $F_2(x)$ measured in the DIS are determined by the quark distributions $xq(x)$, where x is the nucleon momentum fraction carried by a quark.

The QCD SR method was applied for investigation of the DIS on the proton. The second moments of the structure functions were obtained in [110] and [111]. The next to leading terms of the asymptotic expansion in powers of k^{-2} were found in [112]. The structure functions $F_2(x)$ at moderate values of x were calculated in [113]. Another presentation of the structure functions was obtained in [114]. We shall employ the approach, developed in [114] since it can be extended for the case of finite density in a natural way. On the other hand, such generalization is the extension of approach described above.

In order to obtain the distribution of the valence quarks, the authors of [114] considered the correlation function G , which describes the system with the quantum numbers of the proton, interacting twice with strongly virtual hard photons

$$G(q, k) = i^2 \int d^4z d^4y e^{i(qz) + i(ky)} \langle 0 | T j(z) H(y, \Delta) \bar{j}(0) | 0 \rangle. \quad (132)$$

Here q and $q + k$ are the momenta of the nucleon in the initial and final states, $k = k_1 - k_2$ is the momentum, transferred to the system by the photon scattering. The incoming (outgoing) photon carries momentum $k_1(k_2)$, interacting with the quarks in the point $y - \Delta/2$ ($y + \Delta/2$). The quark-photon interaction is described by the function $H(y, \Delta)$.

In [114] the correlation function $G(q, k)$ was calculated in terms of the QCD condensates. The double dispersion relation in variables $q_1^2 = q^2$ and $q_2^2 = (q + k)^2$ was employed. The crucial point was the OPE in terms of nonlocal operators depending on the light-like vector Δ ($\Delta^2 = 0$). The Borel transform in q_1^2 and q_2^2 was carried out. The equal Borel masses $M_1^2 = M_2^2$ are considered. The Fourier transform in Δ provided the momentum distribution of the valence quarks.

Thus, in [114] the momentum distributions of the valence quarks were expressed in terms of the QCD condensates.

Before discussing the extension for nuclear matter, we describe the most intriguing problem of the DIS on nucleus.

5.3.2 EMC effect

The experimental data obtained first by the EMC collaboration [115] showed that the DIS function $F_2^A(x)$ of nucleus with the atomic number A differs from the sum of those of free nucleons. The structure function was compared to that of deuteron ($A = 2$), which imitates the system of free nucleons. The deviation of the ratio

$$R^A(x) = \frac{F_2^A(x)}{A} / \frac{F_2^2(x)}{2} \quad (133)$$

from unity characterizes the deviation of a nucleus from the system of free nucleons.

The ratio $R^A(x)$ appeared to be the function of x indeed. Most of the early data were obtained for iron (Fe). Exceeding unity at $x < 0.2$, the ratio drops at large x , reaching the minimum value $R^{56} \approx 0.85$ at $x \approx 0.7$. This behavior was called EMC effect. The same tendency was traced for other nuclei. Both experimental [116] and theoretical [117] investigations of the effect are going on nowadays.

There are several mechanisms which may cause the deviation of $R^A(x)$ from unity. We shall try to find how the difference of the quark distributions inside the in-medium and free nucleon changes the ratio $R^A(x)$.

5.3.3 EMC effect in QCD sum rules

In [40] the approach of [114] was combined with that of [36] for calculation of the quark distributions in the proton, placed into the nuclear matter. The expression for the correlation function took the form of Eq. (132), with the averaging over vacuum replaced by that over the ground state of the matter. The two types of contributions to the correlator were considered – Fig. 14a,b. In the diagram of Fig. 14a. the photon interacted with the quark of the free loop. In the diagram of Fig. 14b. it interacts with the quark, exchanging with the matter. The modification of the distribution of the quarks was expressed in terms of the vector condensate, with its nonlocal structure being included, and through the shift of the scalar condensate. The results are true only for moderate values of x . They can not be extended to the region $x \ll 1$, since the OPE diverges in that region [113].

Omitting the details of calculation, provided in [40], we present the results in Fig. 15. We carry out calculations for nuclear matter. The ratio $R(x)$ can be viewed as the limiting value

of $R^A(x)$ at $A \rightarrow \infty$. One can see that the distributions of u and d quarks in fraction of the target momentum x are modified in a different way. The fraction of the momentum carried by u quarks decreases by about 4%. The ratio R , determined by Eq. (133) has a typical EMC shape.

Note that while for a free nucleon $x \leq 1$, for the nucleus it can be as large as $x = A$. The developed approach enables to calculate the quark distributions at $x > 1$, describing thus the cumulative aspects of the problem.

5.3.4 Swelling of the nucleon

One of the possible explanations of the EMC effect is the “swelling” of the nucleon inside the nucleus. Under this assumption the structure function of the in-medium nucleon $F_2^m(x)$ is written in terms of that for a free nucleon $F_2^0(x)$ as $F_2^m(x) = F_2^0(xr_m/r_0)$, with $r_m < r_0$ [118]. On the other hand, the value of the proton residue λ^2 is proportional to the square of the three-quark wave function at the origin. Assuming that there is only one length scale r for the nucleon, we find $\lambda^2 \sim r^{-6}$. Employing Eq. (105) we find $(r_m - r_0)/r_0 = 2\%$, in agreement with estimations of [119].

6 Intermediate summary

6.1 Reasons for optimism

One can see that we have some reasons for optimism. The QCD sum rules analysis confirmed that the nucleon in nuclear matter can be treated as moving in superposition of strong vector and scalar fields of the order of about 300 MeV. The vector field is positive, while the scalar field is negative, and a partial cancelation takes place. We obtained this result without employing a controversial conception of interaction of the point-like nucleons.

We obtained a picture for formation of the nucleon self energies. The effective mass m^* is formed by the exchange of noninteracting quarks between the system of the three noninteracting quarks described by the current j_1 determined by Eq.(13) which carries the proton quantum numbers and the scalar condensate, which differs from that in vacuum. Of course, there is strong interactions between the quarks, which form the condensate. The vector self energy Σ_V is formed as the exchange by noninteracting quarks between the system of the three noninteracting quarks, carrying the proton quantum numbers and the valence quarks of the matter, forming the vector condensate. The quarks, which form the condensate interact strongly between themselves and with the other quarks of the matter.

The vector and scalar self-energies are calculated in terms of the in-medium vector and scalar condensates, the nonlocality of the vector condensate is included. The vector condensate can be calculated easily. The contributions, corresponding to the nonlocal structure of the vector condensate can be presented in terms of the moments of the structure functions. Thus, it is also related to observables. The in-medium scalar condensate in the gas approximation, which has a good accuracy near the saturation point can be related to the observable pion-nucleon Σ term. Hence, the nucleon self-energies are expressed in terms of the condensates, which can be either calculated in a model-independent way or related to observables.

The approach was used for calculation of other nucleon parameters, providing reasonable results.

6.2 Reasons for scepticism

However, the results, described above leave some reasons for scepticism. The most important one is the obscure role of the four-quark ($4q$) condensates. A simple estimation for the value of the scalar $4q$ condensate is $\langle M|\bar{q}q\bar{q}q|M\rangle = 2\langle 0|\bar{q}q|0\rangle\langle N|\bar{q}q|N\rangle\rho$. Employing this estimation one would find both vector and scalar self-energies to be large (hundreds MeV) and positive. This would contradict to the QHD phenomenology.

Another obscure point is the role of the radiative corrections. As we have seen in Sec.2, in the lowest order radiative correction to the vacuum SR the coupling constant α_s is multiplied by a numerically large coefficient. The role of radiative corrections in the vacuum case was clarified in [34]. However, the case of finite densities require a separate analysis.

Some of the results obtained in [120] are sometimes viewed as the grounds for additional sceptical statements. One of them is a possible strong shift of the hadron mass m_h due to the resonances in the Nh system, which is not included to the SR analysis. Another one is about the importance of the large distances for the formation of the nucleon mass, while the SR actually deal with small distances of the order 1 GeV^{-1} .

6.3 Response to the sceptical remarks

Start with the latter statement. The nucleon wave function is indeed formed at large distances from the center of nuclei. The case of deuteron is a bright example. However, the NN interactions take place at much smaller distances. One can obtain a rather good description of the deuteron by employing the potential $V(r) = C\delta(r)$. In Walecka model [1] the NN interaction radius is of the order of ω meson inverse mass $1/m_\omega$. The duality interval (27), where the SR are used, just corresponds to the distances responsible for the NN interactions. Hence, the SR approach should provide an adequate description of the nucleon in nuclear matter.

Resonances in Nh system are described by singularities in variable s of the function $\Pi_m(q^2, s)$, defined by Eq. (38). We avoided these singularities by considering $\Pi_m(q^2, s)$ at fixed value of s .

Contribution of the $4q$ condensates and those of the radiative corrections should be included into our analysis. This will be done in next Sections. Note that the calculations of the scalar $4q$ condensate in nucleon in framework of the Nambu–Jona–Lasinio model [121] provided encouraging results. The estimation, which we gave in the beginning of this subsection overshoot the value of condensate since there are certain cancelations.

7 Four-quark condensates

7.1 General equations for contribution of the four-quark condensates

We shall need the expectation values

$$T^{XY,f_1,f_2} = \langle M | : \bar{q}^{f_1 a} \Gamma^X q^{f_1 a'} \bar{q}^{f_2 b} \Gamma^Y q^{f_2 b'} : | M \rangle (\delta_{aa'} \delta_{bb'} - \delta_{ab'} \delta_{ba'}), \quad (134)$$

where the colon signs denote the normal ordering of quark operators, $f_{1,2}$ stand for the quark flavors, a, a', b, b' are the color indices. A nonzero contribution to the function Π_m is provided by the antisymmetric combination of colors - see Eq. (13). The basic 4×4 matrices $\Gamma^{X,Y}$, acting on the Lorentz indices of the quark operators are

$$\Gamma^S = I; \quad \Gamma^{Ps} = \gamma_5; \quad \Gamma^V = \gamma_\mu; \quad \Gamma^A = \gamma_\mu \gamma_5; \quad \Gamma^T = \frac{i}{2}(\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu). \quad (135)$$

Thus, they describe the scalar, pseudoscalar, vector, pseudovector (axial) and tensor cases correspondingly.

In order to simplify the formulas, we shall not display the color indices, keeping in mind that the quark operators are color-antisymmetric.

The contribution of the four-quark condensates to the LHS of the sum rules is [32]

$$(\Pi_m - \Pi_0)_{4q} = (\Pi_\rho)_{4q} = \frac{1}{q^2} \left(\sum_{X,Y} \mu_{XY} H^{XY} + \sum_{X,Y} \tau_{XY} R^{XY} \right). \quad (136)$$

Here

$$H^{XY} = \langle M | \bar{u} \Gamma^X u \bar{u} \Gamma^Y u | M \rangle; \quad R^{XY} = \langle M | \bar{d} \Gamma^X d \bar{u} \Gamma^Y u | M \rangle, \quad (137)$$

while

$$\begin{aligned} \mu_{XY} &= \frac{\theta_Y}{16} \text{Tr}(\gamma_\alpha \Gamma^X \gamma_\beta \Gamma^Y) \gamma_5 \gamma^\alpha \hat{q} \gamma^\beta \gamma_5; \\ \tau_{XY} &= \frac{\theta_Y}{4} \text{Tr}(\gamma_\alpha \hat{q} \gamma_\beta \Gamma^Y) \gamma_5 \gamma^\alpha \Gamma^X \gamma^\beta \gamma_5. \end{aligned} \quad (138)$$

The factor $\theta_Y = 1$ if Γ^X has a vector or tensor structure, in other cases $\theta_Y = -1$. The sign is determined by that of the commutator between the charge conjugation matrix and Γ_Y .

Considering the $4u$ condensates, one can see that the contributions $\mu_{XY} H^{XY}$ to Π_ρ^{4q} obtain nonzero values only if the matrices Γ_X and Γ_Y have the same Lorentz structure. All the structures, presented by Eq. (135) contribute to the $4u$ condensate. In the case of $2d2u$ condensate the factor $\text{Tr}(\gamma_\alpha \hat{q} \gamma_\beta \Gamma^Y)$ does not turn to zero only if Γ_Y has a vector or axial structure. In the latter case Γ_X should have axial structure as well. In the former case it can be either Lorentz scalar or Lorentz vector.

7.2 Approximations for the four-quark condensates

We shall make certain approximations for the quark condensates both in vacuum and in nuclear medium.

7.2.1 Factorization of the vacuum condensate

The vacuum $4q$ condensate will be considered in framework of the factorization hypothesis [20]

$$\langle 0 | \bar{u} \Gamma^X u \bar{u} \Gamma^Y u | 0 \rangle = \langle 0 | \bar{u} \Gamma^X u | 0 \rangle \langle 0 | \bar{u} \Gamma^Y u | 0 \rangle. \quad (139)$$

Thus, on this approximation the condensate $\langle 0 | \bar{u} \Gamma^X u \bar{u} \Gamma^Y u | 0 \rangle$ does not vanish only in the scalar case $\Gamma^X = \Gamma^Y = I$.

$$\langle 0 | \bar{u} u \bar{u} u | 0 \rangle = \langle 0 | \bar{u} u | 0 \rangle \langle 0 | \bar{u} u | 0 \rangle. \quad (140)$$

This approximation has been justified in the limit of large number of colors [93]. This is a standard approximation in the SR calculations. As it stands now, there are no indications of noticeable violation of the factorization relations Eqs. (139), (140).

7.2.2 Gas approximation

The modification of the $4q$ condensates in nuclear matter will be treated in the gas approximation. We shall go beyond in Subsection 7.5. In the gas approximation expectation value of any operator X is

$$\langle M | X | M \rangle = \langle 0 | X | 0 \rangle + \rho \langle N | X | N \rangle, \quad (141)$$

where

$$\langle N | X | N \rangle = \int d^3x \left(\langle N | X(x) | N \rangle - \langle 0 | X(x) | 0 \rangle \right), \quad (142)$$

is the excess of the density $X(x)$ over the vacuum value inside the nucleon. Of course, $\langle 0 | X(x) | 0 \rangle$ does not depend on x .

Assuming that at any space point x

$$\langle N | \bar{u} \Gamma^X u \bar{u} \Gamma^X u | N \rangle = (\langle N | \bar{u}(x) \Gamma^X u(x) | N \rangle)^2,$$

one can write Eq. (142) for $4u$ condensate in another way

$$\begin{aligned} \langle N | \bar{u} \Gamma^X u \bar{u} \Gamma^X u | N \rangle &= \int d^3x \left(\langle N | \bar{u}(x) \Gamma^X u(x) | N \rangle - \right. \\ &\quad \left. - \langle 0 | \bar{u}(x) \Gamma^X u(x) | 0 \rangle \right)^2 + 2 \langle 0 | \bar{u} \Gamma^X u | 0 \rangle \langle N | \bar{u} \Gamma^X u | N \rangle + \\ &\quad + V_N \left((\langle 0 | \bar{u} u | 0 \rangle)^2 - \langle 0 | \bar{u} u \bar{u} u | 0 \rangle \right). \end{aligned} \quad (143)$$

Here V_N is the volume of the nucleon. The last term vanishes under the vacuum factorization assumption – see Eq. (140). For all structures but the scalar one the second term vanishes, since $\langle 0 | \bar{u} \Gamma^X u | 0 \rangle = 0$, and Eq. (143) takes the form

$$\langle N | \bar{u} \Gamma^X u \bar{u} \Gamma^X u | N \rangle = \int d^3x \left(\langle N | \bar{u}(x) \Gamma^X u(x) | N \rangle - \langle 0 | \bar{u}(x) \Gamma^X u(x) | 0 \rangle \right)^2. \quad (144)$$

However, for the scalar case $\Gamma_X = I$ we can write

$$\langle N | \bar{u} u \bar{u} u | N \rangle = \int d^3x \left(\langle N | \bar{u}(x) u(x) | N \rangle - \langle 0 | \bar{u} u | 0 \rangle \right)^2 + 2 \langle 0 | \bar{u} u | 0 \rangle \langle N | \bar{u} u | N \rangle. \quad (145)$$

In similar way we can write for the $2d2u$ scalar-vector condensate, which contributes to the RHS of Eq. (138) for τ_{SV}

$$\begin{aligned} \langle N | \bar{d} d \bar{u} \gamma_0 u | N \rangle &= \int d^3x \left(\langle N | \bar{d}(x) d(x) | N \rangle - \langle 0 | \bar{d} d | 0 \rangle \right) \times \\ &\times \langle N | \bar{u}(x) \gamma_0 u(x) | N \rangle + 2 \langle 0 | \bar{d} d | 0 \rangle \langle N | \bar{u} \gamma_0 u | N \rangle. \end{aligned} \quad (146)$$

In the gas approximation we can write for the condensates, defined by Eq. (137)

$$\begin{aligned} H^{XY} &= h_p^{XY} \rho_p + h_n^{XY} \rho_n; \quad h_N^{XY} = \langle N | \bar{u} \Gamma^X u \bar{u} \Gamma^Y u | N \rangle \\ R^{XY} &= r^{XY} \rho; \quad r^{XY} = \langle N | \bar{d} \Gamma^X d \bar{u} \Gamma^Y u | N \rangle. \end{aligned} \quad (147)$$

Further calculations require certain quark model of nucleon.

7.3 Perturbative Chiral Quark Model

7.3.1 Description of the model

The Perturbative Chiral Quark Model (PCQM) originally suggested in [30], was developed later in [122]. Further applications are reviewed in [123]. The nucleon is considered as a system of three relativistic valence quarks, moving in an effective static field. The valence quarks are supplemented by a cloud of pseudoscalar mesons, introduced in agreement with the requirements of the chiral symmetry. In the $SU(2)$ version of the model, which will be used here, only the pions are included. The meson cloud is included in the lowest order of perturbation theory.

The PCQM Lagrangian is the sum of the terms, describing the constituent quarks, pions and their interaction

$$\begin{aligned} L &= L_Q + L_\pi + L_{int}; \quad L_Q = \bar{\psi}(r) [i\hat{\partial} - S(r) - \gamma_0 V(r)] \psi(r); \\ L_\pi &= \frac{1}{2} (\partial_\mu \phi_i(r))^2; \quad L_{int} = -i\bar{\psi}(r) S(r) \gamma_5 \frac{\tau^i \phi_i(r)}{f_\pi} \psi(r). \end{aligned} \quad (148)$$

Here ϕ_i is the isotriplet of the pion fields. The contribution L_{int} is the lowest order expansion of the chiral interaction term $L_{int} = -\bar{\psi}(r) S(r) e^{i\gamma_5 \tau^i \phi_i(r)/f_\pi} \psi(r)$, f_π is the pion decay constant. We did not write down the quark mass term.

Important steps in development of the model were made in [124]. In earlier applications the pions were considered as independent point-like degrees of freedom. In [124] they were treated as containing the “sea” quarks ($\bar{q}q$ pairs) of the nucleon.

Another move touched the treatment of the constituent quarks. The authors of [124] did not solve the Dirac equation for given form of the potentials $S(r)$ and $V(r)$, but postulated the Gaussian shape of the constituent quark density. It was assumed that the coordinate part of wave function of the constituent quark can be represented as the product of three single-particle functions

$$\psi(r) = \phi(r) \chi(r); \quad \phi(r) = N e^{-r^2/2R^2}; \quad \chi(r) = \begin{pmatrix} \chi_0 \\ i\beta \frac{(\sigma r)}{R} \chi_0 \end{pmatrix} \quad (149)$$

with the normalization condition $\int d^3r \bar{\psi}(x) \gamma_0 \psi(x) = 1$ expressing the conservation of the baryon charge. Parameters of the model β and R are fitted to reproduce the values of the axial coupling constant and of the proton charge radius correspondingly. Employing the Dirac equation one finds that the wave function represented by Eq. (149) corresponds to the scalar field

$$S(r) = M + cr^2,$$

where $M = (1 - 3\beta^2)/2\beta R \approx 230 \text{ MeV}$ can be treated as constituent mass of the quark.

7.4 Four-quark condensates in the PCQM

Now each of the condensates h_N^{XY} and r^{XY} defined by Eq. (147) can be presented as consisting of three contributions. All the four operators can act on the valence quarks. This will be denoted by the lower index *val*. All the four operators can act on the pions. This will be denoted by the lower index *P*. There is also a possibility that two operators act on the constituent quarks while the other two act on pions. Such interference terms will be denoted by the lower index *J*. Hence, we can write

$$h_N^X = (h_N^X)_{val} + (h^X)_P + (h^X)_J; \quad r^{XY} = (r^{XY})_{val} + (r^{XY})_P + (r^{XY})_J. \quad (150)$$

Since in the case of $4u$ condensate only the structures with $X = Y$ contribute to the SR, we used notation $h^{XX} = h^X$. In the analysis presented in the next subsection we omit several terms, which are numerically not important. Start with the contribution of the valence quarks.

7.4.1 Contribution of the valence quarks

Here the products of the quark operators are averaged over the state $|\tilde{N}\rangle$ of three constituent quarks, described by the wave functions presented by Eq. (149). Due to the normal ordering of the quark operators we find for neutrons $(h_n^X)_{val} = 0$, while for protons

$$(h_p^X)_{val} = \langle U | \bar{u} \Gamma^X u \bar{u} \Gamma^X u | U \rangle.$$

Here $|U\rangle$ denotes the vector of state of two constituent U quarks.

For the scalar case the product of the PCQM operators describes the excess of the quark density with respect to the vacuum value

$$\begin{aligned} \bar{u}^{PCQM}(r) u^{PCQM}(r) &= \bar{u}(r) u(r) - \langle 0 | \bar{u} u | 0 \rangle; \\ \langle U | \bar{u}^{PCQM} u^{PCQM} | U \rangle &= 2 \int d^3r \bar{\psi}(r) \psi(r). \end{aligned} \quad (151)$$

Thus, employing Eqs. (142), (143) we find for the scalar case $X = Y = S$

$$(h_p^S)_{val} = 2 \langle 0 | \bar{u} u | 0 \rangle \langle \tilde{N} | \bar{u} u | \tilde{N} \rangle + \int d^3r (\bar{\psi}(r) \psi(r))^2. \quad (152)$$

The last factor $\langle \tilde{N} | \bar{u} u | \tilde{N} \rangle$ in the first term on the RHS has the meaning of the contribution of the valence quarks to parameter κ_N defined by Eq. (4). In the PCQM model $\langle \tilde{N} | \bar{u} u | \tilde{N} \rangle = 2 \int d^3r \bar{\psi}(r) \psi(r) = 1.08$.

For the other structures of $4u$ condensate we have just

$$(h_p^X)_{val} = \int d^3r (\bar{\psi}(r) \Gamma^X \psi(r))^2. \quad (153)$$

For the $2d2u$ condensate we can write

$$(r_N^{XY})_{val} = \langle D | d \Gamma^X d | D \rangle \langle U | \bar{u} \Gamma^Y u | U \rangle.$$

Here $|D\rangle$ is the vector of state of constituent D quark. Analysis, similar to that, made for the $4u$ condensates provides for the scalar-vector case $X = S, Y = V$

$$(r^{SV})_{val} = 2 \langle 0 | \bar{d} d | 0 \rangle \langle \tilde{N} | \bar{u} \gamma_0 u | \tilde{N} \rangle + 2 \int d^3r \bar{\psi}(r) \psi(r) \bar{\psi}(r) \gamma_0 \psi(r). \quad (154)$$

Note that $\langle \tilde{N} | \bar{u} \gamma_0 u | \tilde{N} \rangle = 2$. For other structures

$$(r^{XX})_{val} = \int d^3r (\bar{\psi}(r) \Gamma^X \psi(r))^2. \quad (155)$$

7.4.2 Contribution of the sea quarks

Now all the quark operators act on the pions. The contribution is expressed in terms of pion expectation values of the four-quark operators

$$(\Pi_{4q})_{pions} = \frac{1}{16} \left(\sum_{X,\alpha} \langle \pi^\alpha | \mu_X \bar{u} \Gamma^X u \bar{u} \Gamma^X u + \tau_X \bar{d} \Gamma^X d \bar{u} \Gamma^X u | \pi^\alpha \rangle \right) \frac{\partial \Sigma^\alpha}{\partial m_\pi^2}. \quad (156)$$

Here α denotes the pion isotopic states, Σ stands for the sum of the self-energy pion loop and the pion exchange contribution. The pion expectation values can be expressed in terms of the vacuum expectation values of the four-quark operators. This was done in [125] by employing the current algebra technique. Using the results of [125] we find

$$\sum_X \langle \pi^\alpha | \mu_X \bar{u} \Gamma^X u \bar{u} \Gamma^X u + \tau_X \bar{d} \Gamma^X d \bar{u} \Gamma^X u | \pi^\alpha \rangle = 0, \quad (157)$$

and thus

$$(\Pi_{4q})_{pions} = 0. \quad (158)$$

Hence, there is no such thing as “the contribution caused by the sea quarks only”. This is true for any model where the sea quarks are contained in the pions. Note that in the calculations of [125] the factorization assumption Eq. (140) have been used.

7.4.3 Contribution of the interference terms

In the case of the scalar condensate the interference terms can be written as

$$(h_p^I)_J = 2 \sum_i \langle \tilde{N} | H_{int} | \tilde{N}_i, \pi \rangle \langle \tilde{N}_i | \bar{u} u | \tilde{N} \rangle \langle \pi | \bar{u} u | \pi \rangle \langle \tilde{N}, \pi | H_{int} | \tilde{N} \rangle, \quad (159)$$

where H_{int} is the interaction between the constituent quark Q and the pion. One can write $H_{int} = -L_{int}$, with L_{int} given by Eq. (148). Here two quark operators act on the pion, while

the other two operators act on a constituent quark. The latter can be the same as one in the matrix element of the interaction H_{int} or another one. Actually in the sum over the states of the constituent quarks \tilde{N}_i only those, corresponding to the nucleon are included, i.e. $|\tilde{N}_i\rangle = |\tilde{N}\rangle$.

In the case of pseudoscalar and axial operators the interference can take place in the first order of the πQ interaction. This happens because the matrix elements $\langle\pi|\bar{q}\Gamma^X q|0\rangle$ have nonzero values in these cases. The contribution of such “vertex interference” is

$$(h_p^X)_J = \langle\tilde{N}|H_{int}|\tilde{N}, \pi\rangle\langle\tilde{N}, \pi|\bar{q}\Gamma^X q\bar{q}\Gamma^X q|\tilde{N}\rangle + \langle\tilde{N}, \pi|\bar{q}\Gamma^X q\bar{q}\Gamma^X q|\tilde{N}\rangle\langle\tilde{N}|H_{int}|\tilde{N}, \pi\rangle, \quad (160)$$

with X labelling an axial or pseudoscalar. Thus in one of the vertices interaction H_{int} is replaced by the amplitude of injection of four quarks and antiquarks. For neutral pions the latter can be evaluated as

$$\begin{aligned} \langle\tilde{N}, \pi^0|\bar{q}\Gamma^X q\bar{q}\Gamma^X q|\tilde{N}\rangle &= \langle\pi^0|\bar{q}\Gamma^X q|0\rangle\langle\tilde{N}|\bar{q}\Gamma^X q|\tilde{N}\rangle, \\ \langle\tilde{N}|\bar{q}\Gamma^X q\bar{q}\Gamma^X q|\tilde{N}, \pi^0\rangle &= \langle 0|\bar{q}\Gamma^X q|\pi^0\rangle\langle\tilde{N}|\bar{q}\Gamma^X q|\tilde{N}\rangle, \end{aligned}$$

with analogous relations for the charged pions.

Now we shall employ the values of the four-quark condensates, obtained in [31] in the SR equations.

8 Contribution of the higher order terms

8.1 Symmetric matter with the four-quark condensates

We can write the general equation for the contribution of the $4q$ terms

$$(\Pi)_{4q} = \left(X_{4q}^q \frac{\hat{q}}{q^2} + X_{4q}^P \frac{(Pq)}{m^2} \frac{\hat{P}}{q^2} + X_{4q}^I \frac{(Pq)}{m^2} \frac{I}{q^2} \right) \frac{a}{(2\pi)^2} \rho. \quad (161)$$

Note that $a = -(2\pi)^2 \langle 0|\bar{u}u|0\rangle \approx 0.55 \text{ GeV}^3$ (Eq. (24)) is just a convenient scale for presentation of the results. It does not reflect the chiral properties of $(\Pi)_{4q}$.

The coefficients X_{4q}^i are obtained by using the complete set of the nucleon four-quark condensates [31] and the results of the previous Section. In calculations of the factor X_{4q}^q all contributions are numerically important due to their partial cancellations. The coefficient X_{4q}^P is determined mainly by the $2d2u$ vector–vector condensate. The factor X_{4q}^I is dominated by the vector–scalar $2d2u$ condensate, with the first term on the RHS of Eq. (154) providing the largest contribution.

The numerical values are

$$X_{4q}^q = -0.11; \quad X_{4q}^P = 0.57; \quad X_{4q}^I = 1.27. \quad (162)$$

We took into account the nonlocal structure of the vector condensate in the factor X_{4q}^I .

Now we estimate the ratio of contributions of the $4q$ condensates to those of the condensates with $d = 3$, provided by Eqs. (65), (66). The effective values of momenta are $|q^2| \sim 1 \text{ GeV}^2$. Putting logarithmic factors in Eqs. (65), (66) equal to unity, we find that this ratio is less than 0.1 in the \hat{q} structure of the QCD sum rules. It is about 0.13 in the \hat{P} structure. The

ratio is about 1/4 in the scalar structure I for $\kappa_N = 8$. It becomes smaller for larger values of κ_N . As we shall see below, the gas approximation result for $(X^I)_{4q}$ provided by Eq. (162) overestimates the value. Thus, the values of contributions of $4q$ condensates are consistent with the assumption on the convergence of the OPE series.

The contributions of dimension $d = 6$ to the sum rules – see Eqs. (57), (64) are

$$\tilde{A}_6 = -8\pi^2 a X_{4q}^q \rho; \quad \tilde{P}_6 = -8\pi^2 a \frac{s - m^2}{2m} X_{4q}^P \rho; \quad \tilde{B}_6 = -8\pi^2 m a X_{4q}^I \rho. \quad (163)$$

Actually, \tilde{B}_6 contains also the terms of the higher dimension, since it includes the nonlocality of the vector condensate.

The medium induced $4q$ condensates on the LHS of the sum rules correspond the exchange by strongly correlated four-quark systems on the RHS. This may be a local two-meson exchange, with two mesons interaction with the nucleon at the same point – see Fig. 16. Another possible interpretation is the exchanges by the four-quark mesons, if there are any [126].

However, the leading contribution to the scalar structure, determined by the first term on the RHS of Eq. (154) has another interpretation. Here two quarks are exchanged with vacuum, while two other ones are exchanged with the valence quarks of the matter. The contribution of this scalar-vector condensate to X_{4q}^I is

$$X_{4q}^{SV} = -\frac{2(Pq)}{3q^2} \frac{\langle 0|\bar{d}d|0\rangle v(\rho)}{m} \quad (164)$$

This term contains the expectation value $\langle 0|\bar{d}d|0\rangle$ as a factor. On the other hand this term is proportional to the vector condensate, contributing, however, to the scalar Lorentz structure of the nucleon equation of motion Eq. (1). On the RHS of the SR this can be interpreted as the vector meson exchange with the anomalous structure of the vertex. We discussed such contributions in the analysis of the charge-symmetry breaking forces in Subsec. 5.2 – see Fig. 13.

Note that if we go beyond the gas approximation, the discussed term contains rather the expectation value $\langle M|\bar{d}d|M\rangle = \kappa(\rho)/2$ instead of the vacuum value $\langle 0|\bar{d}d|0\rangle$. As we see from Eq. (92), $|\kappa(\rho)/\kappa(0)| < 1$, with the reduction of about 30 – 50% at saturation density. Hence, the gas approximation overestimates strongly the value of X_{4q}^I . In the next to leading order beyond the gas approximation we must consider the contribution, in which two pairs of quarks of the $4q$ condensate go to two different nucleons. Such terms should be considered together with the other three-body contributions.

As we have seen in Sec. 4, the nucleon parameters depend on the nucleon expectation value κ_N , which is known with large uncertainties. We show density dependence of the nucleon parameters for $\kappa_N = 8$ obtained in [32] in Fig. 17. In Fig. 18 we present the dependence of nucleon self-energies on κ_N at the saturation value of density.

For $\kappa_N = 8$ we find at $\rho = \rho_0$

$$\Sigma_V = 160 \text{ MeV}; \quad m^* - m = -380 \text{ MeV}. \quad (165)$$

Thus, the $4q$ condensates subtract 90 MeV from the vector self-energy and 170 MeV from the effective mass m^* , provided by Eq. (105). The potential energy $U = -180$ MeV looks discouraging. However, inclusion of the radiative corrections improves the situation.

8.2 Radiative corrections

The radiative corrections to the vector condensate, calculated recently in [127] appeared to be rather large. Thus we have to analyze the role of the radiative corrections more carefully.

In Subsec. 2.3 we clarified the role of these corrections in vacuum SR. Here we focus on the terms Π_ρ , provided by medium. The contributions of the condensates of the lowest dimension $d = 3$, provided by Eq. (66) are actually written in the form

$$\tilde{A}_{3\rho} = \frac{\tilde{A}_{3\rho}^{(0)}}{L^{\gamma_q}}; \quad \tilde{B}_{3\rho} = \frac{\tilde{B}_{3\rho}^{(0)}}{L^{\gamma_I}}; \quad \tilde{P}_{3\rho} = \frac{\tilde{P}_{3\rho}^{(0)}}{L^{\gamma_P}}, \quad (166)$$

where the anomalous dimensions $\gamma_q = \gamma_P = 4/9, \gamma_I = 0$. Here the upper index 0 denotes that all radiative corrections are neglected. The factors $L^{-\gamma_i}$ include the sum of the terms $(\alpha_s \ln q^2)^n$. This is called the Leading Logarithmic Approximation (LLA). Actually, in our previous analysis we included the contributions of condensates with $d = 3$ with the radiative corrections treated in LLA. Note that in computation of the next to leading order terms we used the structure functions [68], which reproduce their moments with proper anomalous dimensions. We did not include the radiative corrections to the $4q$ condensates, since these terms were obtained in framework of a model, which does not contain gluons. Also, in comparison with the steep q^2 dependence of the terms, containing the $4q$ condensates, caused by the “normal” high dimension, the role of anomalous dimension is relatively small.

Now we shall include the corrections of the order α_s beyond the LLA. We present

$$\tilde{A}_{3\rho} = \tilde{A}_{3\rho}^{(0)} t_a; \quad \tilde{B}_{3\rho} = \tilde{B}_{3\rho}^{(0)} t_b; \quad \tilde{P}_{3\rho} = \tilde{P}_{3\rho}^{(0)} t_P. \quad (167)$$

Here

$$t_i = \frac{r_i}{L^{\gamma_i}}, \quad (168)$$

with

$$r_i = 1 + c_i \alpha_s / \pi,$$

where the second term on the RHS is the lowest order radiative correction beyond the LLA. While $c_I = 3/2$ [54], it was found in [127] that $c_q = 7/2$ and $c_P = 15/4$. Thus, employing a straightforward estimation, we can expect a 40 – 50% change of the nucleon parameters due to radiative corrections.

We shall compare the three cases. All the radiative corrections are ignored, i.e. all $t_i = 1$. Corrections are included in framework of LLA, i.e. all $r_i = 1$. The third possibility is that in addition to LLA the lowest order α_s correction is included beyond the LLA.

We carry out the computations for $\kappa_N = 8$, assuming $\alpha_s(1 \text{ GeV}) = 0.37$. The results of calculations are presented in Table 1. We see the LLA corrections subtracts 70 and 50 MeV from the vector self-energy and the effective mass correspondingly. Inclusion of the corrections beyond the LLA makes the values of Σ_V and m^* closer to those obtained with total neglect of the radiative corrections. The results are illustrated by Fig. 19.

Another problem is the dependence of numerical results on the actual value of α_s . The latter is determined by the value of Λ_{QCD} . The authors of [127] used $\alpha_s(1 \text{ GeV}) = 0.47$, while $\alpha_s(1 \text{ GeV}) = 0.55$ was employed in [22]. These values corresponds to $\Lambda_{QCD} = 0.23 \text{ GeV}$ and $\Lambda_{QCD} = 0.28 \text{ GeV}$. These variations of α_s change the nucleon self-energies by several MeV, affecting mostly the value of the nucleon residue [35].

8.3 Asymmetric matter

8.3.1 Inclusion of condensates of higher dimension

The nonlocal structure of the vector condensate is included in the same way as in the case of the symmetric matter.

Employing the values of the four-quark condensates obtained in the previous Section we find the β dependence of the contributions of the four-quark condensates, defined by Eq. (161)

$$X_{4q}^q = -0.11 - 0.21\beta; \quad X_{4q}^P = 0.57 + 0.09\beta; \quad X_{4q}^I = 1.27 - 0.61\beta. \quad (169)$$

Including also the nonlocal structure of the vector condensates, we solve the SR equations. In the actual numerical calculations we employ the PCQM value $\zeta_p = 0.54$ for the isotope asymmetric condensate of dimension $d = 3$ – see Eq. (110). In Fig. 20, we show the values of the proton and neutron self-energies in neutron matter ($\beta = 1$), compared to those in symmetric matter. In other words, these are the proton self-energy values for the nuclear matter with $\beta = -1, 0, 1$ [33].

One can see that the β dependence of self-energies has the same qualitative features as that determined by Eq. (112). In the matter with neutron excess $\beta > 0$ the neutron vector self-energy is larger than the proton one. The proton effective mass is larger than the neutron one. In both cases the β dependence is relatively weak.

8.3.2 Comparison with results of nuclear physics

Our results for the difference of the effective masses of neutron and proton $m_{np}^* = m_n^* - m_p^*$ appears to be twice smaller than the result of [128], but only 30% smaller than that of [129]. However, a discrepancy from the relativistic Brueckner–Hartree–Fock (RHFB) calculations [129] is smaller. Their results for $\beta = 0.2$ are $\Sigma_V^n - \Sigma_V^p = 30$ MeV and $m_{np}^* = -15$ MeV. We obtained 20 MeV and -15 MeV correspondingly for these parameters. Another RHFB calculation [130] provided results, which are very close to ours one. Their values for $\beta = 0.75$ are $\Sigma_V^n - \Sigma_V^p = 80$ MeV and $m_{np}^* = -50$ MeV, while we found 80 MeV and -55 MeV correspondingly.

Note that the nuclear physics calculations provide $m_{np}^* < 0$ for $\beta > 0$. Our values have the same sign after the four-quark condensates are included. The lowest dimension solution expressed by Eq. (112) provides $m_{np}^* > 0$.

We found unexpectedly satisfactory agreement for the potential energy splitting $U_{np} = U^{(n)} - U^{(p)}$ and even for the β dependence of the average binding energy per nucleon $\varepsilon(\rho, \beta)$. Various approaches [131, 132] provided $U_{np} \approx 60$ MeV, while our result is $U_{np} = 40$ MeV. The “symmetry energy” $\varepsilon^{sym} = 1/2 \partial \varepsilon(\rho_0, \beta) / \partial \beta$ is 29 MeV in our approach. All earlier [128]–[132] and nowadays [133] calculations provide $\varepsilon^{sym} \approx 30$ MeV. We did not expect a good agreement here, since these parameters contain the large terms, which cancel each other to large extent.

8.4 Many-body interactions

The structure and the role of the three-body (many-body) nuclear interactions is much studied nowadays – see, e.g. [18, 134, 135]. In the SR approach such interactions emerge in a natural way.

There are two types of many-body interactions in our approach. One of them is connected with the four-quark condensate. Another one manifests itself in the baryon-hole excitations, taken into account by the in-medium pion propagator.

In Sec. 6 the contribution of four-quark condensates was included in the gas approximation. Our probe nucleon exchanged both pairs of quarks with the same nucleon of the matter. Beyond the gas approximation two quark pairs can be exchanged with different nucleons of the matter. This corresponds to the three-body interactions.

Such terms are shown in Fig. 21. Their contribution, e.g. to the LHS of Eq. (57) for the q structure $\mathcal{L}_m^q(M^2, W_m^2)$ can be estimated as $4\pi^4\kappa_N^2\rho^2$. At the saturation value of density this can change the value of $\mathcal{L}_m^q(M^2, W_m^2)$ by about 10%. This will lead to relative changes of the same order of the nucleon self-energies. At larger values of density the role of these terms increases.

As we discussed in Sec. 3, the contributions, involving $n \geq 2$ nucleons of the matter have the branching points in variable $S_n(q^2) = (nP + q)^2$. Generally speaking, this means that we need a more complicated model of the spectrum. However, we see that in the OPE series the three body terms do not contain branching points in q^2 . In framework of our model for the higher lying states they contribute only to the pole term on the RHS of the sum rules.

Adding gluon interactions with the matter, i.e. including the four-quark-gluon condensates, we obtain the terms, involving larger number of nucleons of the matter, corresponding to many-nucleon forces. They contribute to the higher order OPE terms. Due to the small value of the in-medium gluon condensate we expect such contributions to be small.

Another type of many-body interactions manifests itself in the nonlinear contribution to the scalar condensate $S(\rho)$. As we saw in Subsection 4.5, it is determined by the pion cloud. The general form of the contribution is

$$S = - \int \frac{d^3p}{(2\pi)^3} \int \frac{d^4k}{i(2\pi)^4} \sum_B \left(\Gamma_B(k) D^{(m)2}(k) G_B(p-k) \Gamma_B(k) - \right. \\ \left. - \Gamma_B^{(0)}(k) D^{(0)2}(k) G_B^{(0)}(p-k) \Gamma_B^{(0)}(k) \right) \langle \pi | \bar{q}q | \pi \rangle, \quad (170)$$

where summation over spin and isospin variables is assumed. Here G_B is the propagator of intermediate baryon in nuclear matter, $D^{(m)}(k)$ is the pion propagator, renormalized by the particle-hole excitations of medium. The vertices of $N\pi B$ interaction in nuclear matter are denoted as Γ_B . Integration over momenta of the nucleons of the matter is limited by condition $p \leq p_F$. Upper index (0) denotes the vacuum functions. The second term on the RHS of Eq. (170) subtracts the terms, which are already included in the physical nucleon. For $D^{(m)} = D^{(0)}$; $\Gamma_B = \Gamma_B^{(0)}$ the RHS of Eq. (170) turns to that of Eq. (115). The contribution is illustrated by Fig. 22.

The pion propagator $D^{(m)}(k)$ includes the multi-nucleon effects. Thus, Eq. (170) includes the many-body interactions. However, the rigorous calculation of its RHS is a part of a self-consistent problem.

9 Self-consistent scenario

We have seen that the shape of the density dependence of the quark condensates $\kappa(\rho)$ is very important for the hadronic physics. It is also important for description of the matter as a whole, since characterizes a degree of restoration of the chiral symmetry with the growing density. On the other hand, we saw that it determines the main density dependence of the effective mass $m^*(\rho) \approx m^*(\kappa(\rho))$.

The linear part of the density behavior of $\kappa(\rho)$ is related to observables and can be calculated in a model-independent way. However, the nonlinear contribution, expressed by Eq. (170) depends on several hadron parameters. If the intermediate baryon in Eq. (170) is a nucleon, these are the nucleon effective mass m^* and the nucleon pion coupling constant $g_{\pi NN}(\rho)$. The latter can be expressed in terms of the pion decay constant $f_\pi(\rho)$ and the nucleon isovector axial coupling $g_A(\rho)$ via the Goldberger–Treiman relation [136]

$$g_{\pi NN}/2m = g_A/f_\pi.$$

The density dependence of g_A and f_π can also be obtained in the SR approach. One should add similar equations for the contribution of delta-isobars to the RHS of Eq. (170). Thus we come to self-consistent set of equations which can be solved within the QCD sum rules approach.

$$m^* = m^*(\kappa); \quad f_\pi = f_\pi(\kappa); \quad g_A = g_A(\kappa); \quad \kappa = \kappa(m^*, f_\pi/g_A). \quad (171)$$

Of course, the SR approach should be combined with some other ones. We have seen already that the calculation of the four-quark condensate required a quark model for the nucleon. Also, the RHS of Eq. (170) contains the in-medium pion propagator. It satisfies the Dyson equation

$$D^{(m)} = D^{(0)} + D^{(0)}\Pi_\pi D^{(m)},$$

where the pion polarization operator Π_π describing the baryon-hole excitations of the matter. The operator Π_π depends strongly on the hadron correlations at the distances $r \geq 1/m_\pi$, corresponding to small momenta. They can be included by employing of the Finite Fermi System Theory (FFST) [137], [138]. Thus, in order to obtain the many-body effects it is reasonable to combine the SR approach with the FFST.

10 Summary

We demonstrated that the QCD sum rules provide a consistent formalism for solving various problems of nuclear physics.

We expressed the nucleon parameters in nuclear matter in terms of the in-medium QCD condensates. In symmetric matter the leading contributions to the vector and scalar self-energies are expressed in terms of vector and scalar condensates. The vector condensate $v(\rho)$ can be calculated easily. In the gas approximation, corresponding to inclusion only of the two-body interactions, the scalar condensate $\kappa(\rho)$ is related to the observable pion–nucleon sigma-term.

In other words, exchange by the strongly correlated quark systems (mesons) is expressed in terms of exchange by the system of weakly interacting quarks with the same quantum numbers.

Thus the nucleon self-energies are obtained without employing a controversial conception of interaction of the point-like nucleons.

Inclusion of the condensates of the lowest dimension confirmed that the nucleon in nuclear matter can be treated as moving in superposition of strong vector and scalar fields of the order of about 300 MeV. The vector field is positive, while the scalar field is negative, and a strong cancellation takes place.

This result is not altered by inclusion of the four-quark condensates. Thus we can expect the convergence of the series with successive inclusion of the condensates of higher dimension. Note, however, that the calculation of the in-medium four-quark condensates requires employing a quark model of nucleon.

We used the approach for calculation of other nucleon characteristics in nuclear matter. We calculated the nucleon self-energies in asymmetric nuclear matter and found their dependence on the difference between the densities of the neutrons and protons. We calculated the in-medium quenching of isovector axial coupling constant. We found the neutron-proton mass difference, providing at least qualitative explanation of the Nolen–Schiffer anomaly.

Also, we demonstrated that our approach enables to investigate internal structure of the nucleon, providing description of some features of the EMC effect. Such problems are inaccessible for nuclear physics.

Inclusion of the nonlinear density dependence of the scalar condensate $\kappa(\rho)$ corresponds to taking into account the many-body interactions. We show that the nonlinear terms in $\kappa(\rho)$ are provided mainly by the pion cloud. A simple inclusion of the contributions beyond the gas approximation signals on a possible saturation mechanism, which is due to the many-body effects. Thus it differs from that in the Walecka model.

A more rigorous treatment of the nonlinear contributions to $\kappa(\rho)$ requires solution of the self-consistent problem, in which the coupling constants $g_{\pi NN}$ and $g_{\pi N\Delta}$ as well as the effective mass of the Δ isobar should be also determined as functions of κ , while κ should be expressed in terms of these parameters. This would provide the dependence of $\kappa(\rho)$. This is an important parameter of the matter itself, since it shows the degree of restoration of the chiral symmetry.

This analysis requires employing of the pion propagator, which is renormalized by the particle-hole excitations of the matter. These excitations can be included by using the Finite Fermi System Theory, formulated and developed by A. B. Migdal and his colleagues several decades ago.

The work on the project is in progress.

11 Epilogue

Many years ago the authors of this paper participated in a very interesting discussion on the perspectives of nuclear physics, which took place at the traditional Petersburg (Leningrad at that time) Winter School of Physics. In his lecture A. B. Migdal formulated two questions, he wished to be answered in the future:

- How do the QCD condensates change in nuclear medium?

- What is the connection between the in-medium QCD condensates and hadron parameters?

We hope that the present paper makes the first steps to answer these questions.

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References

- [1] J. D. Walecka, *Ann. Phys.* **83**, 491 (1974).
- [2] B. D. Serot and J. D. Walecka, *Adv. Nucl. Phys.* **16**, 1 (1985).
- [3] L. S. Celenza and C. M. Shakin, *Relativistic Nuclear Physics*, World Scientific, Philadelphia, 1986.
- [4] R. D. Furnstahl and B. D. Serot, *Phys. Rev. C* **47**, 2338 (1993).
- [5] M. Stoitsov, *et al.*, *J. of Physics, Conference Series*, **180**, 012082 (2009).
- [6] J. Dobaczewski, arXiv:1009.0899 v1 [nucl-th] 5 Sept. 2010.
- [7] J. W. Negele, *Comm. Nucl. Part. Phys.* **14**, 1 (1985).
- [8] L. A. Sliv, M. I. Strikman and L. L. Frankfurt, *Sov. Phys.-Uspekhi*, **28**, 281 (1985).
- [9] M. Ericson, A. Figureau, and C. Thevenet, *Phys. Lett B* **45**, 19 (1973).
- [10] M. Rho, *Nucl. Phys. A* **231**, 493 (1974).
- [11] J. A. Miller, B. M. K. Nefkens, and I. Slaus, *Phys. Rep.* **194**, 1 (1990).
- [12] P. A. M. Guichon, *Phys. Lett. B* **200**, 235 (1988).
- [13] K. Saito and A. W. Thomas, *Phys. Rev. C* **51**, 2757 (1995).
- [14] T. Tsushima, K. Saito, A. W. Thomas and A. Valcarce, *Eur. Phys. J. A* **31**, 626 (2007).
- [15] F. Myhrer, J. Wroldsen, *Rev. Mod. Phys.* **60**, 629 (1988).
- [16] S. Weinberg, *Nucl. Phys. B* **363**, 3 (1991).
- [17] E. Epelbaum, H. W. Hammer and Ulf-G. Meißner, *Rev. Mod. Phys.* **81**, 1773 (2009).
- [18] R. Machleidt, arXiv: nucl-th/0609050, nucl-th/07040807;
R. Machleidt and D. R. Eutem, *J. Phys. G* **37**, 064041 (2010).
- [19] B. L. Ioffe, V. S. Fadin and L. N. Lipatov, *Quantum Chromodynamics*, Cambridge Univ. Press, 2010.
- [20] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, *Nucl. Phys. B* **147**, 385 (1979).
- [21] B. L. Ioffe, *Nucl. Phys. B* **188**, 317 (1981); **E 191**, 591 (1981).
- [22] B. L. Ioffe, *Prog. Part. Nucl. Phys.* **56**, 232 (2006).
- [23] B. L. Ioffe, arXiv:0810.4234; *Phys. At. Nucl.* **72**, 1214 (2009).

- [24] E. G. Drukarev and E. M. Levin, Pis'ma ZhETF. **48**, 307 (1988); (JETP Lett. **48**, 338 (1988)); ZhETF **95**, 1178 (1989) (Sov. Phys. JETP **68**, 680 (1989).
- [25] E. G. Drukarev and E. M. Levin, Nucl. Phys. A **511**, 679 (1990).
- [26] E. G. Drukarev and E. M. Levin, Prog. Part. Nucl. Phys. **27**, 77 (1991).
- [27] T. P. Cheng, Phys. Rev. D **13**, 2161 (1976).
- [28] T. P. Cheng and R. Dashen, Phys. Rev. Lett. **26**, 594 (1971).
- [29] J. Gasser, H. Leutwyler and M. E. Sainio, Phys. Lett. B **253**, 252, 260 (1991).
- [30] Th. Gutsche and D. Robson, Phys. Lett. B **229**, 333 (1989).
- [31] E. G. Drukarev, M. G. Ryskin, V. A. Sadovnikova, V. E. Lyubivitskij, Th. Gutsche, and A. Faessler, Phys. Rev. D **68**, 054021 (2003).
- [32] E. G. Drukarev, M. G. Ryskin, V. A. Sadovnikova, Th. Gutsche, and A. Faessler, Phys. Rev. C **68**, 065210 (2004).
- [33] E. G. Drukarev, M. G. Ryskin and V. A. Sadovnikova, Phys. Rev. C **70**, 065206 (2004).
- [34] V. A. Sadovnikova, E. G. Drukarev and M. G. Ryskin, Phys. Rev. D **72**, 114015 (2005).
- [35] E. G. Drukarev, M. G. Ryskin and V. A. Sadovnikova, Phys. Rev. C **80**, 045208 (2009).
- [36] E. G. Drukarev and M. G. Ryskin, Nucl. Phys. A **578**, 333 (1994).
- [37] E. G. Drukarev, M. G. Ryskin and V. A. Sadovnikova, Eur. Phys. J. **4**, 171 (1999).
- [38] E. G. Drukarev and E. M. Levin, Nucl. Phys. A **532**, 695 (1991).
- [39] E. G. Drukarev and M. G. Ryskin, Nucl. Phys. A **572**, 560 (1994); A **577**, 375c (1994).
- [40] E. G. Drukarev and M. G. Ryskin, Z. Phys. A **356**, 457 (1997).
- [41] L. J. Reinders, H. R. Rubinstein, and S. Yazaki, Phys. Rep. **127**, 1 (1985).
- [42] K. G. Wilson, Phys. Rev. **179**, 499 (1969).
- [43] B. L. Ioffe, Z. Phys. C **18**, 67 (1983).
- [44] B. L. Ioffe and A. V. Smilga, Nucl. Phys. B **232**, 109 (1984).
- [45] M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. **175**, 2195 (1968).
- [46] A. I. Vainshtein, V. I. Zakharov and M. A. Shifman, JETP Letters, **27**, 55 (1978).
- [47] A. E. Dorohov, N. I. Kochelev, Z. Phys. C **46**, 281 (1990).
- [48] H. Forkel, M. K. Banerjee, Phys. Rev. Lett. **71**, 484 (1993).

- [49] V. A. Sadovnikova, E. G. Drukarev and M. G. Ryskin, *Yad. Phys.* **71**, 1459 (2008) [*Phys. At. Nucl.* **71**, 1431 (2008)].
- [50] E. G. Drukarev, M. G. Ryskin and V. A. Sadovnikova, *Phys. Rev. D* **80**, 014008 (2009).
- [51] Y. Chung, H. G. Dosch, M. Kremer and D. Schall, *Phys. Lett. B* **102**, 175 (1981); *Nucl. Phys. B* **197**, 55 (1982).
- [52] H. G. Dosch, M. Jamin and S. Narison, *Phys. Lett. B* **220**, 251 (1989).
- [53] M. E. Peskin, *Phys. Lett. B* **88**, 128 (1979).
- [54] A. A. Ovchinnikov, A. A. Pivovarov, and L. R. Surguladze, *Int. J. Mod. Phys. A* **6**, 2025 (1991).
- [55] M. Jamin, *Z. Phys. C* **37**, 635 (1988).
- [56] K. Nakamura, *et al.* (Particle Data Group), *J. Phys. G* **37**, 075502 (2010).
- [57] D. B. Leinweber, *Ann. Phys. (NY)* **254**, 328 (1997).
- [58] E. M. Henley and J. Pasupathy, *Nucl. Phys. A* **556**, 467 (1993).
- [59] H. Lehmann, *Nuovo Cimento*, **11**, 342 (1954).
- [60] A. A. Abrikosov, L. P. Gorkov, and I. E. Dzyaloshinskii, “*Methods of Quantum Field Theory in Statistical Physics*”, Prentice-Hall, Inc., Englewood Cliffs, N.J. 1963.
- [61] R. J. Furnstahl, D. K. Grigel and T. D. Cohen, *Phys. Rev. C* **46**, 1507 (1992).
- [62] X. Jin, M. Nielsen, T. D. Cohen, R. J. Furnstahl and D. K. Grigel, *Phys. Rev. C* **49**, 464 (1994).
- [63] X. Jin, *Phys. Rev. C* **51**, 2260 (1995).
- [64] X. Jin and R. J. Furnstahl, *Phys. Rev. C* **49**, 464 (1994).
- [65] T. D. Cohen, R. J. Furnstahl, D. K. Grigel and X. Jin, *Prog. Part. Nucl. Phys.* **35**, 221 (1995).
- [66] C. J. Horowitz and B. D. Serot, *Nucl. Phys. A* **464**, 613 (1987).
- [67] C. Itzykson and J.-B. Zuber, “*Quantum Field Theory*”, (McGraw-Hill, NY, 1980).
- [68] A. Glück, E. Reya and A. Vogt, *Eur. Phys. J. C* **5**, 461 (1998).
- [69] Yo-xin Liu, Dong-fen Gao, and Hua Guo, *Phys. Rev. C* **68**, 035204 (2003).
- [70] E. G. Drukarev, M. G. Ryskin and V. A. Sadovnikova, *Prog. Part. Nucl. Phys.* **47**, 73 (2001).

- [71] J. Gasser and M. E. Sainio, hep-ph/0002283;
M. E. Sainio, hep-ph/0110413; π N Newsletter **16**, 138 (2002).
- [72] R. Koch, Z. Phys. C **15**, 161 (1982).
- [73] M.M. Pavan, I. I. Strakovsky, R. L. Workman, and R. A. Arndt, π N Newsletter **16**, 110 (2002).
- [74] P. Schweitzer, Eur. Phys. J. A **22**, 89 (2004).
- [75] G. E. Hite, W. B. Kaufmann, and R. J. Jacob, Phys. Rev. C **71**, 065201 (2005).
- [76] E. Reya, Rev. Mod. Phys. **46**, 545 (1974).
- [77] M. Procura, Th. R. Hemmert and W. Weise, Phys. Rev. D **69**, 034505 (2004).
- [78] C. C. Barros Jr, and M. B. Robilotta, Eur. Phys. J. C **45**, 445 (2006).
- [79] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Phys. Lett. B **78**, 443 (1978).
- [80] K. Gottfried, Phys. Rev. Lett. **18**, 1174 (1967).
- [81] A. L. Kataev, arXiv: hep-ph/0311091.
- [82] S. Weinberg, in “*A Festschrift for I. I. Rabi*”, L. Motz, ed. (NY, 1977).
- [83] M. Anselmino and S. Forte, Z. Phys. C **61**, 453 (1994).
- [84] S. Forte, Phys. Rev. D **47**, 1842 (1993).
- [85] A. Bulgac, G. A. Miller, and M. I. Strikman, Phys. Rev. C **56**, 3307 (1997).
- [86] B. L. Birbrair and E. L. Kryshen, Yad. Phys. **72**, 1092 (2009); [Phys. At. Nucl. **72**, 1154 (2009)].
- [87] M. Lutz, D. Friman, Ch. Appel, Phys. Lett. B **474**, 7 (2000).
- [88] E. G. Drukarev, M. G. Ryskin and V. A. Sadovnikova, Z. Phys. A **353**, 455 (1996).
- [89] F. Tondeur, Nucl. Phys. A **303**, 185 (1978).
- [90] V. M. Belyaev and Ya. I. Kogan, Phys. Lett. B **136**, 273 (1984).
- [91] C. B. Chiu, J. Pasupathy and S. J. Wilson, Phys. Rev. D **32**, 1786 (1985).
- [92] E. M. Henley, W-Y. P. Hwang and L. S. Kisslinger, Phys. Rev. D **46**, 431 (1992).
- [93] V. A. Novikov, M. A. Shifman, A. I. Vainshtein, M. B. Voloshin and V. I. Zakharov, Nucl. Phys. B **237**, 525 (1984).
- [94] B. Parthasarathy and J. Pasupathy, Phys. Rev. D **37**, 2140 (1988).

- [95] D. H. Wilkinson, Phys. Rev. C **7**, 930 (1973).
- [96] G. D. Alkhazov, S. A. Artamonov, V. I. Isakov, K. A. Mezilev and Yu. N. Novikov, Phys. Lett. B **198**, 37 (1987).
- [97] S. Shlomo, Rep. Prog. Phys. **41**, 957 (1978).
- [98] J. A. Nolen, J. P. Schiffer, Annu. Rev. Nucl. Part. Sci. **19**, 471 (1969).
- [99] E. M. Henley and G. Krein, Phys. Rev. Lett. **62**, 2586 (1989).
- [100] U. G. Meißner and H. Weigel, Phys. Lett. **267**, 167 (1991).
- [101] K. Saito and A. W. Thomas, Phys. Lett. B **335**, 17 (1994).
- [102] U. G. Meißner, A. M. Rakhimov, A. Wirzba and U. T. Yakshiev, Eur. Phys. J. A **31**, 357 (2007).
- [103] U. G. Meißner, A. M. Rakhimov, A. Wirzba and U. T. Yakshiev, Eur. Phys. J. A **36**, 37 (2008).
- [104] D. Espir, P. Pascual and R. Tarrach, Nucl. Phys. B **214**, 285 (1983).
- [105] C. Adami, E. G. Drukarev, and B. I. Ioffe, Phys. Rev. D **48**, 2304 (1993).
- [106] K. C. Yang, W.-Y. P. Hwang, E. M. Henley, and L. S. Kisslinger, Phys. Rev. D **47**, 3001 (1993).
- [107] T. Hatsuda, H. Høgaasen and M. Prakash, Phys. Rev. Lett. **66**, 2851 (1991).
- [108] C. Adami and G. Brown, Z. Phys. A **340**, 93 (1991).
- [109] T. Schafer, V. Koch, and G. Brown, Nucl. Phys. A **562**, 644 (1993).
- [110] A. V. Kolesnichenko, Sov. J. Nucl. Phys. **39**, 968 (1982).
- [111] V. M. Belyaev and B. Yu. Blok, Z. Phys. C. **30**, 279 (1986).
- [112] V. M. Braun and A. V. Kolesnichenko, Nucl. Phys. B **283**, 723 (1987).
- [113] V. M. Belyaev and B. L. Ioffe, Nucl. Phys. B **310**, 548 (1988).
- [114] V. Braun, P. Gornicki, and L. Mankiewicz, Phys. Rev. D **51**, 6036 (1995).
- [115] J. J. Aubert, Phys. Lett. B **123**, 275 (1983).
- [116] J. Seely *et al.*, Phys. Rev. Lett. **103**, 202301 (2009).
- [117] D. W. Higinbotham, J. Gomez, E. Piasetsky, ArXiv: 1003.4497 v2 [hep-ph].
- [118] R. L. Jaffe, Phys. Rev. Lett. **50**, 228 (1983).

- [119] L. Frankfurt and M. Strikman, Phys. Rep. **160**, 235 (1988).
- [120] V. L. Eletsky and B. L. Ioffe, Phys. Rev. Lett. **78**, 1010 (1997).
- [121] L. S. Celenza, C. M. Shakin, W. D. Sun, and J. Szweda, Phys. Rev. C **51**, 937 (1995).
- [122] V. E. Lyubovitskij, Th. Gutsche, and A. Faessler, Phys. Rev. C **64**, 065203 (2001).
- [123] Th. Gutsche, V. E. Lyubovitskij, and A. Faessler, Prog. Part. Nucl. Phys. **50**, 235 (2003).
- [124] V. E. Lyubovitskij, Th. Gutsche, A. Faessler, and E. G. Drukarev, Phys. Rev. D **63**, 054026 (2001).
- [125] E. G. Drukarev, M. G. Ryskin, V. A. Sadovnikova, and A. Faessler, Phys. Rev. D **65**, 074015 (2002); 66, 039903 (E).
- [126] D. P. Roy, J. Phys. G **30**, R113 (2004).
- [127] S. Groote, J. G. Körner, and A. A. Pivovarov, Phys. Rev. D **78**, 034039 (2008).
- [128] V. Greco, M. Colonna, M. Di Toro, G. Fabbri, and E. Matera, Phys. Rev. C **64**, 045203 (2001).
- [129] F. de Jong and H. Lenske, Phys. Rev. C **57**, 3099 (1998).
- [130] H. Huber, F. Weber, and M. K. Weigel, Phys. Rev. C **51**, 1790 (1995).
- [131] I. Bombaci and U. Lombardo, Phys. Rev. C **44**, 1892 (1991).
- [132] W. Zuo, I. Bombaci and U. Lombardo, Phys. Rev. C **60**, 024605 (1999).
- [133] Chen Lie Wen, arhiv: 0910.008 v1 [nucl-th].
- [134] Huang Dong, T. T. S. Kuo, and R. Machleidt, Phys. Rev. C **80**, 065803 (2009).
- [135] K. Hebeler and A. Schwenk, Phys. Rev. C **82**, 014314 (2010).
- [136] M. L. Goldberger and S. B. Treiman, Phys. Rev. **110**, 1478 (1958).
- [137] A. B. Migdal, *Theory of Finite Fermi Systems and Application to Atomic Nuclei*, Wiley, NY, 1969.
- [138] A. B. Migdal, D. N. Voskresensky, E. E. Saperstein, M. A. Troitsky, Phys. Rep. **192**, 179 (1990);
A. B. Migdal, D. N. Voskresensky, E. E. Saperstein, M. A. Troitsky, *Pion degrees of freedom in nuclear medium*, Nauka, Moscow, 1991.

Table 1: Nucleon parameters at the saturation value of nucleon density. Line 1: all radiative corrections are neglected. Line 2: radiative corrections are included in LLA. Line 3: corrections $\sim \alpha_s$ are included beyond the LLA (BLLA).

	Σ_v (MeV)	$m^* - m$ (MeV)	λ_m^2 (GeV ⁶)	W_m^2 (GeV ²)
No corrections	229	-329	1.10	1.72
LLA	160	-380	1.10	1.77
BLLA	271	-300	1.41	1.75

Table 2: Dependence of the nucleon parameters on the values of Λ_{QCD} and α_s at the phenomenological saturation value of nucleon density with radiative corrections included beyond the leading logarithmic approximation (BLLA). The results are presented for $\Lambda_{\text{QCD}} = 0.23 \text{ GeV}$, $\alpha_s(1 \text{ GeV}^2) = 0.47$, and $\Lambda_{\text{QCD}} = 0.28 \text{ GeV}$, $\alpha_s(1 \text{ GeV}^2) = 0.55$. The corresponding values of vacuum parameters are given in brackets.

α_s (1 GeV ²)	Σ_v (MeV)	$m^* - m$ (MeV)	λ_m^2 (GeV ⁶)	W_m^2 (GeV ²)
0.47	269	-291($m = 0.93 \text{ GeV}$)	1.65(2.35)	1.89(2.13)
0.55	264	-289($m = 0.94 \text{ GeV}$)	1.81(2.61)	1.99(2.26)

Figure Captions

Fig. 1. Main contributions to the LHS of the vacuum sum rules – Eq. (19). Helix line stands for the correlator, solid lines denote the quarks, dotted lines stand for the quarks composing the scalar condensate. Figs. *a*, *b* and *c* correspond to the contributions A_0 , B_3 and A_6 .

Fig. 2. A typical radiative correction. The dashed line denotes the gluon.

Fig. 3. Contributions to the singularities of the correlator in *s* channel (*a*) and in *u* channel (*b*).

Fig. 4. Self-energy insertions to the nucleon pole contributions: the mean field approximation (*a*), the self-energy in the direct channel (*b*) and the exchange contribution (*c*).

Fig. 5. One of the contributions, containing a higher lying singularity in q^2 at fixed *s*. Helix line stands for the correlator. Dashed lines stand for the mesons, solid line is for the nucleon and thick the solid line denotes the nuclear matter.

Fig. 6. A singularity in *u* channel, containing an intermediate state with the baryon number $B = 0$. The meaning of the lines is the same as in Fig. 5.

Fig. 7. Contributions of the condensates of lowest dimension to the left-hand side of the sum rules in nuclear matter. The dotted lines in (*a*) stand for the quarks of the scalar condensate. The dashed–dotted lines in (*b*) denote the quarks of the vector quark condensate. The meaning of the other lines is the same as in previous figures.

Fig. 8. Contributions of the condensates of lowest dimension to the right-hand side of the sum rules in nuclear matter. The horizontal line denotes the nucleon, thick lines show the matter. The wavy and dashed lines stand for vector and scalar mesons.

Fig. 9. The dependence of the functions f_q , f_P and f_I defined by Eq. (81) on the Borel mass M^2 .

Fig. 10. Density dependence of the nucleon parameters for $\kappa_N = 8$: the vector self-energy and effective mass (*a*); the residue λ_m^2 and the threshold W_m^2 (*b*), related to their vacuum values. The *x* axis corresponds to the density, related to its saturation value. Solid line is for solution of Eq.(57) with condensates of dimensions $d = 3$ and $d = 4$ taken into account. Dashed lines show the approximate solution, corresponding to Eqs. (89)–(91).

Fig. 11. Dependence of the nucleon parameters on the value of κ_N at saturation density. Solid line is for the effective mass m^* , dotted line shows the vector self-energy Σ_V , dashed and dash–dotted lines are for the λ_m^2 and W_m^2 correspondingly. All parameters are related to their vacuum values (the vector self-energy is related to the vacuum value of the nucleon mass) given by Eq.(29).

Fig. 12. Simplest contribution to the nonlinear scalar condensate. Solid lines labeled as 1 and 2 are the nucleons of the matter. Dashed line denotes a π meson.

Fig. 13. Anomalous Lorentz structure of nucleon interactions with matter. Notation are the same as in Fig. 8.

Fig. 14. Second order interaction of the hard photon (dashed lines) with the correlator. The dark blobs denote interaction with the matter. Other notations are the same as in Fig. 5.

Fig. 15. The in-medium changes of the *d* quark distribution (dashed curve) and of the *u* quark distribution (dot-dashed curve) of the fraction *x* of the momentum of the target nucleon. The solid curve presents the function $R - 1$ with the ratio *R*, defined by Eq. (126).

Fig. 16. Interactions on the RHS of the sum rules, corresponding to inclusion of the four-quark condensates. The wavy lines denote mesons.

Fig. 17. Density dependence of the nucleon parameters for $\kappa_N = 8$ with the condensates of higher dimensions included: the vector self-energy and effective mass (a); the residue λ_m^2 and the threshold W_m^2 (b), related to their vacuum values. The x axis corresponds to the density, related to its saturation value.

Fig. 18. Dependence of the nucleon parameters on the value of κ_N at saturation density. Condensates of higher dimension are included. The meaning of the curves is the same as that for Fig. 11.

Fig. 19. Density dependence of the vector self energy Σ_V and of the scalar self energy $m^* - m$. The nuclear matter density ρ is related to its saturation value ρ_0 . Dotted lines: all radiation corrections are neglected. Dashed lines: radiative corrections are included in the Leading Logarithm Approximation (LLA). Solid lines: Corrections of the order α_s are included perturbatively beyond the LLA.

Fig. 20. The density dependence of the vector self-energy Σ_V (a) and of the effective mass m^* (b) in isospin asymmetric matter. The solid line is for symmetric matter. The dashed and dotted lines are for the proton and neutron characteristics in neutron matter ($\beta = 1$).

Fig. 21. A contribution of the three-body forces to the LHS of the sum rules. Two pairs of quarks from the four-quark condensate interact with two different nucleons of the matter, shown by the dotted circles.

Fig. 22. Interaction of the operator $\bar{q}q$ (the dark blob) with pion field. The solid line denotes the nucleon of the matter; the wavy line stands for the pion. The bold wavy line denotes the pion propagator renormalized by the baryon-hole excitations.

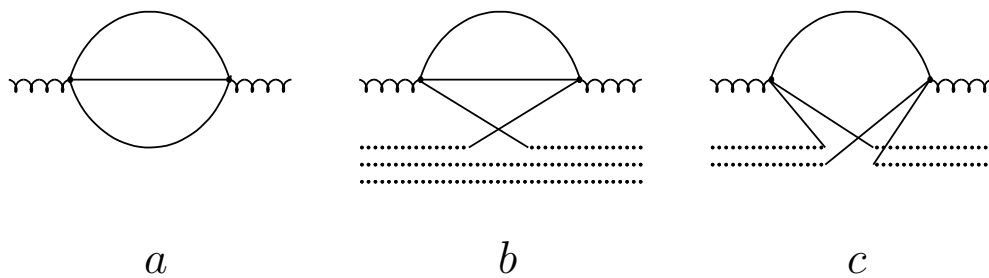


Figure 1:

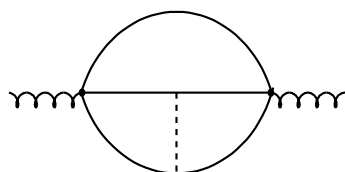


Figure 2:

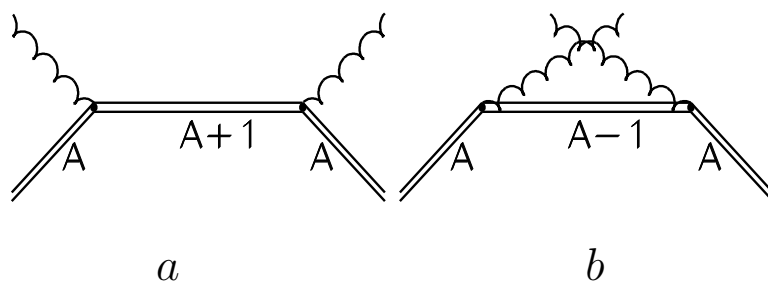


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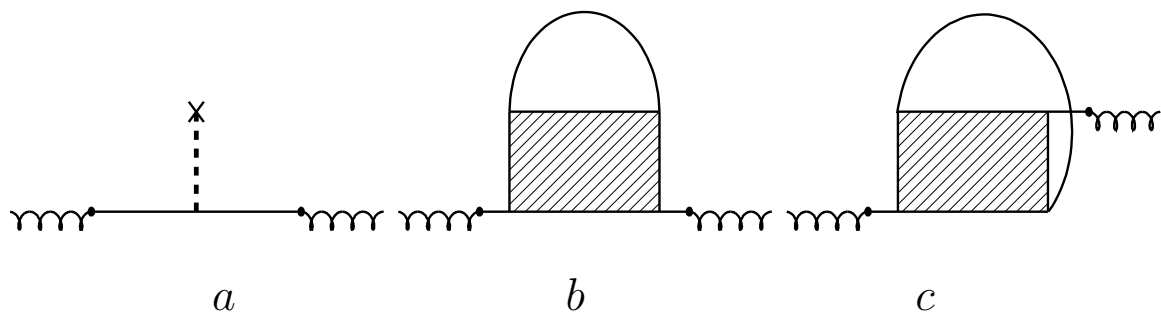


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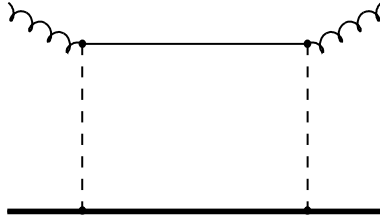


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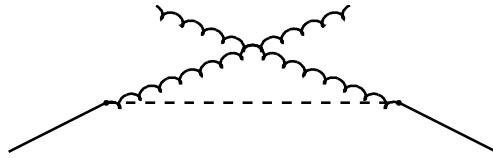


Figure 6:

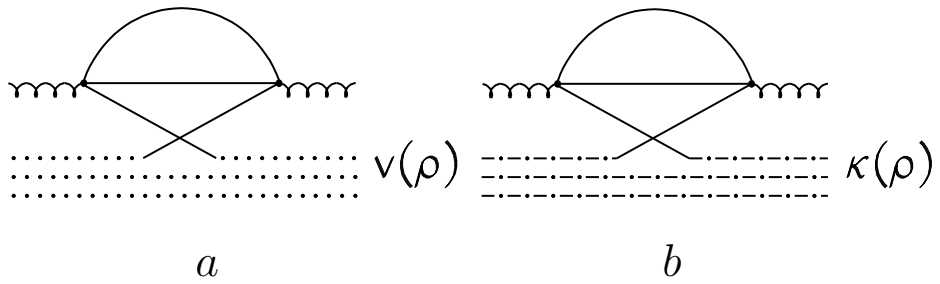


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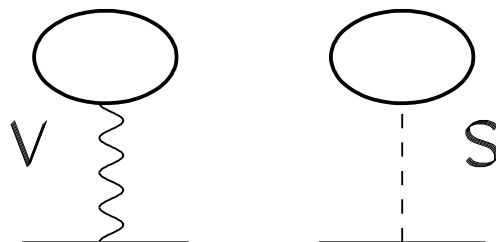


Figure 8:

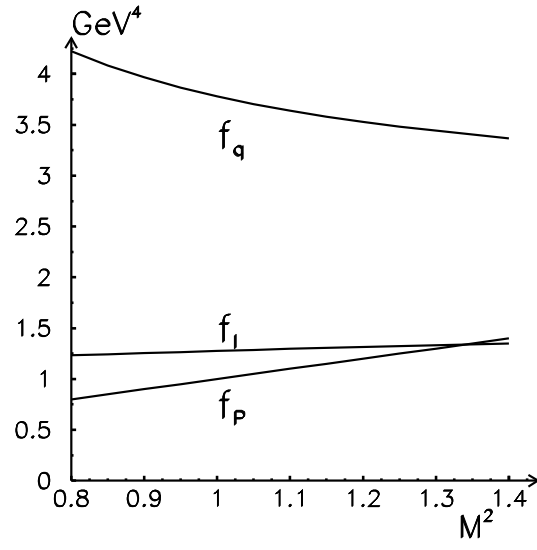


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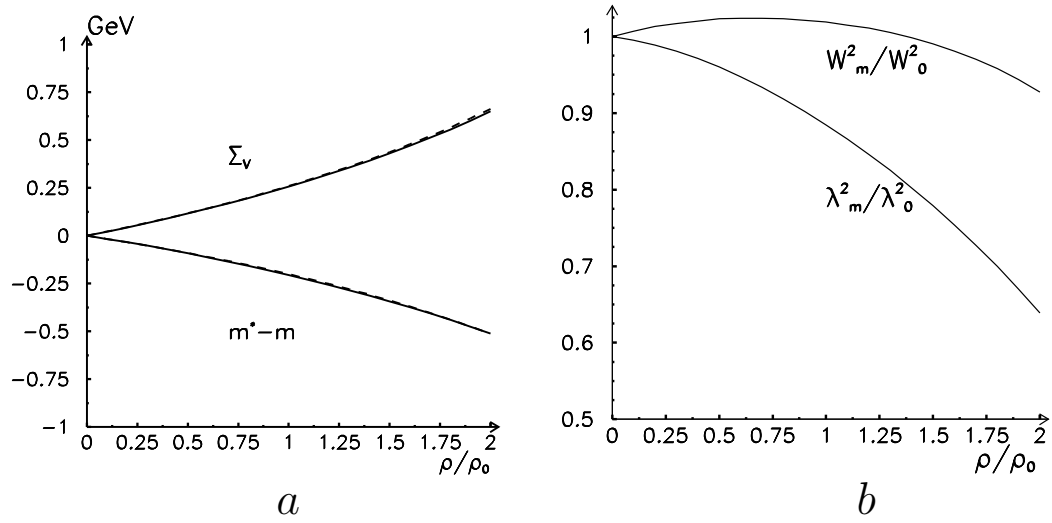


Figure 10:

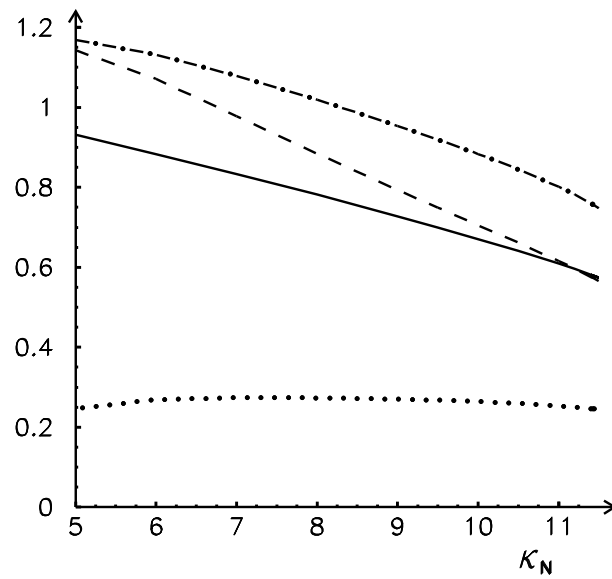


Figure 11:

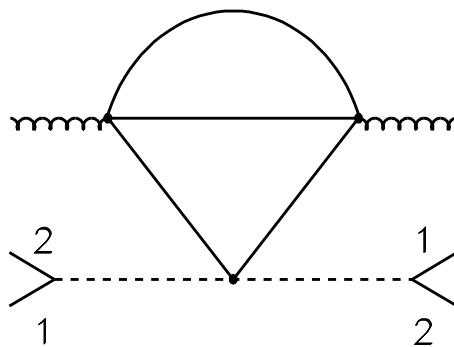


Figure 12:

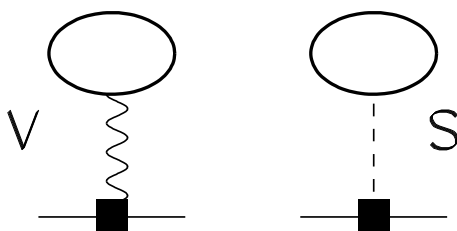


Figure 13:

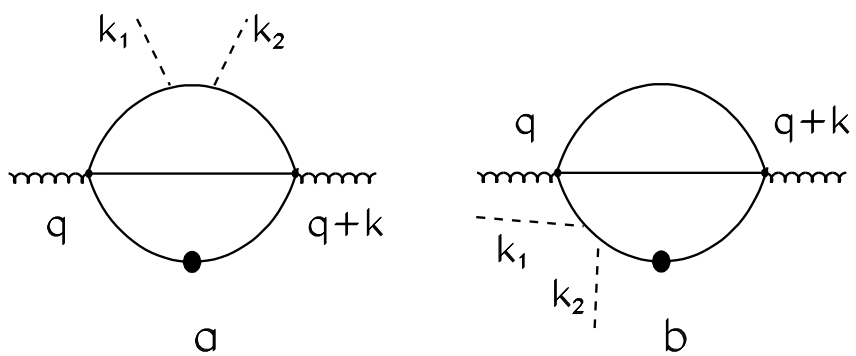


Figure 14:

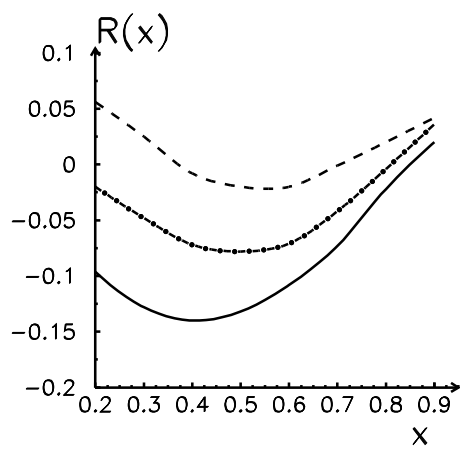


Figure 15:

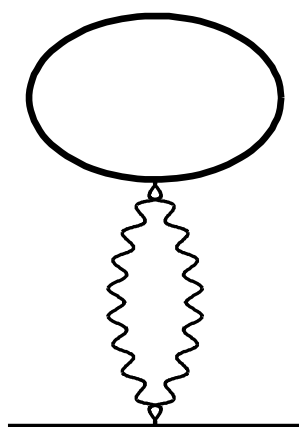


Figure 16:

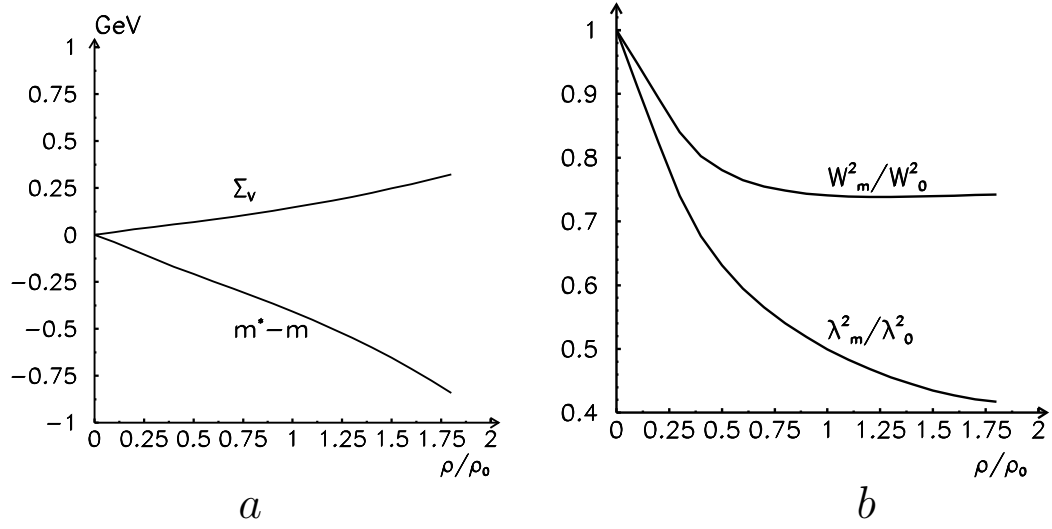


Figure 17:

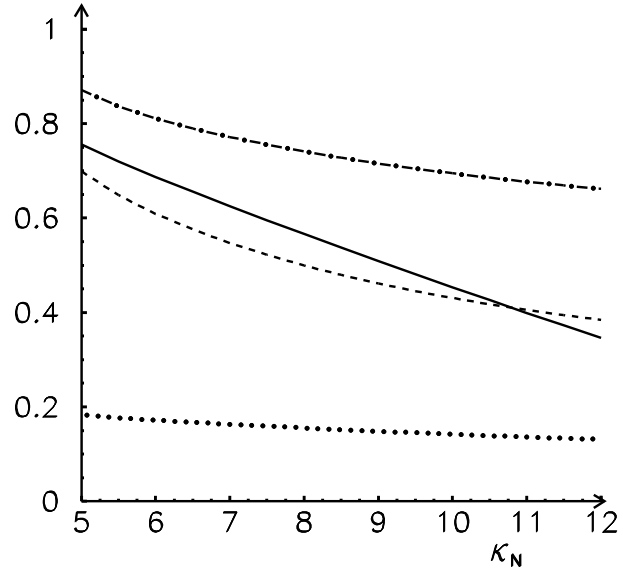


Figure 18:

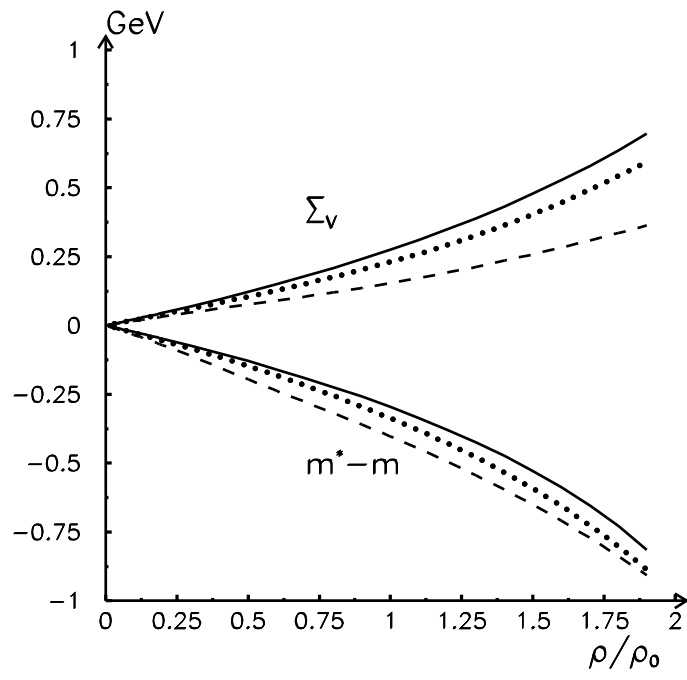


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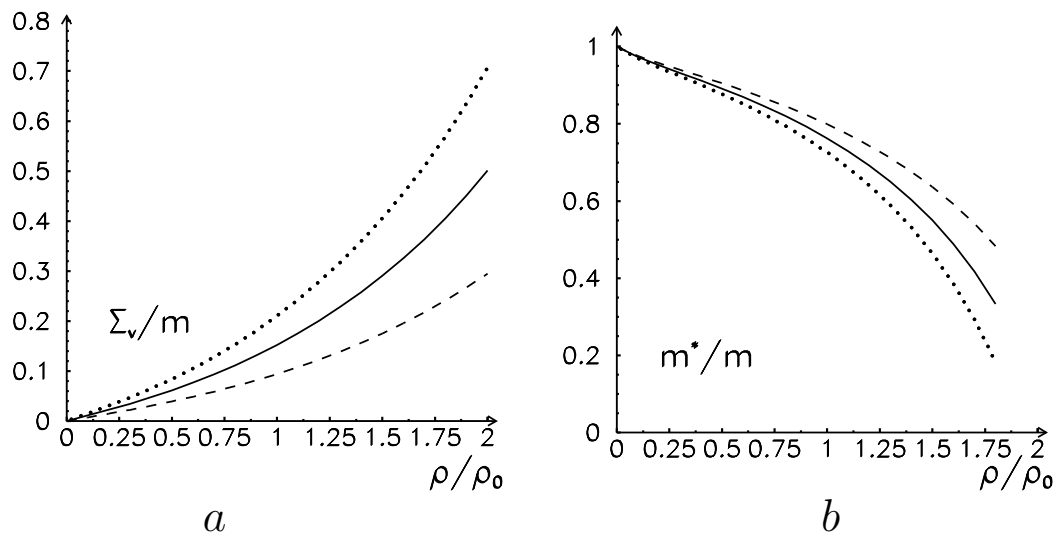


Figure 20:

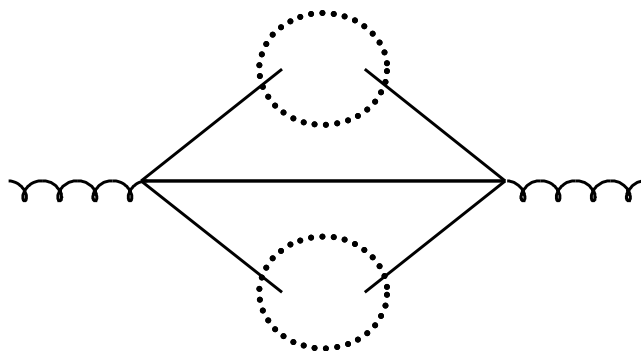
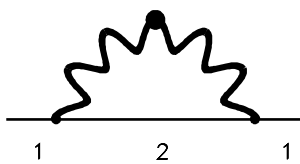


Figure 21:



$$\text{wavy line} = \text{wavy line} + \text{wavy line} \bigcirc \text{wavy line}$$

Figure 22: